CMR
INSTITUTE OF
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Internal Assesment Test - II

Sub:	ENGINEERING MATHEMATICS III							Code:	15MAT31
Date:	02 / 11 / 2016	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	CV-A,B, IS-B MECH-A

First question (8 marks) is compulsory. Answer any six questions from the rest.(6×7=42)

		Marks	OBE CO	OBE RBT
1	Solve $u_{n+2}-2u_{n+1}+u_n=2^n$, $u_0=2, u_1=1$ by using Z transforms	[08]	CO2	L3
2	Find Z transform of $\cos n\theta\, and \sin n\theta$ and hence deduce Z transform of $e^{-an}\cos(n\theta)\ and\ e^{-an}\sin(n\theta)$.	[07]	CO2	L3
3	Find Fourier transform of $f(x)=\begin{Bmatrix} 1, & \lvert x\rvert\le 1 \\ 0, & \lvert x\rvert>1\end{Bmatrix}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$	[07]	CO2	L3
4	Find the inverse Fourier sine transform of $F_s(\alpha)=\frac{1}{\alpha}e^{-a\alpha}, a>0$	[07]	CO2	L3
5	Fit a second degree parabola $y=ax^2+bx+c$ for the given data by the method of least squares.	[07]	CO6	L3

X	1	2	3	4	5
Y	10	12	13	16	19

		Marks	CO	RBT
6	Fit an exponential curve of the form $y=ae^{bx}$ by the method of least squares for the following data.	[07]	CO6	L3

No.of petals	5	6	7	8	9	10
No.of flowers	133	55	23	7	2	2

		Marks	CO	RBT
7	Obtain the lines of regression and hence find the coefficient of correlation for the following data .	[07]	CO4	L3

X	1	2	3	4	5	6	7
Y	9	8	10	12	11	13	14

		Marks	CO	RBT
8	If $\vec{f}=(x^2-27)\vec{i}-6yz\vec{j}+8xz^2\vec{k}$ evaluate $\int_c \vec{f}.d\vec{r}$ from O(0,0,0) to P(1,1,1) along the straight line from O to A(1,0,0) , A to B(1,1,0) and B to P.	[07]	CO5	L3

Solution - Internal II (2.11.2016)

CV - A, B, IS A, B
Mech - A.

1) $u_{n+2} - 2u_{n+1} + u_n = 2^n$; $u_0 = 2$; $u_1 = 1$

Taking z Transform on both side

$z_T(u_{n+2}) - 2z_T(u_{n+1}) + z_T(u_n) = z_T(2^n)$ 2m

$z^2\left(\bar{U}(z) - u_0 - \frac{u_1}{z}\right) - 2z\left(\bar{U}(z) - u_0\right) + z_T(u_n) = \frac{z}{z-2}$

$\Rightarrow \bar{U}(z)(z^2 - 2z + 1) - 2z^2 - z + 4z = \frac{z}{z-2}$ (using $u_0 = 2$; $u_1 = 1$)

$\Rightarrow \bar{U}(z) = \frac{z}{(z-2)(z-1)^2} + \frac{z(2z-3)}{(z-1)^2}$

let $\bar{U}(z) = P(z) + Q(z)$ — ① 2m

Now $P(z) = \frac{z}{(z-2)(z-1)^2} = \frac{Az}{(z-2)} + \frac{Bz}{(z-1)} + \frac{Cz}{(z-1)^2}$

$1 = A(z-1)^2 + B(z-1)(z-2) + C(z-2)$

Put $z = 1 \Rightarrow C = -1$; put $z = 2 \Rightarrow A = 1$ & put $z = 0$; get B

$\therefore P(z) = \frac{z}{(z-2)} - \frac{z}{(z-1)} - \frac{z}{(z-1)^2}$ — ②

$q(z) = \frac{z(2z-3)}{(z-1)^2} = \frac{Az}{(z-1)} + \frac{Bz}{(z-1)^2} = \frac{2z}{(z-1)} - \frac{z}{(z-1)^2}$ — ③

(put $z = 1 \Rightarrow B = -1$ & comparing coeff of z^2 we get $A = 2$)

$\therefore$ eq ① becomes $z_T(u_n) = \frac{z}{(z-2)} + \frac{z}{(z-1)} - \frac{2z}{(z-1)^2}$ 3m

Taking z inverse we get

$u_n = 2^n - 1^n - 2n$ 1m

// soln.

2. $e^{in\theta} = \cos n\theta + i\sin n\theta$

$$(e^{i\theta})^n = (e^{i\theta})^n = k^n$$

$$k = e^{i\theta}$$

$$Z_T[k^n] = \frac{z}{z-k}$$

$$Z_T[e^{in\theta}] = \frac{z}{z-e^{i\theta}} = \frac{z(z-e^{-i\theta})}{(z-e^{-i\theta})(z-e^{i\theta})}$$

$$= \frac{z[z-(\cos\theta - i\sin\theta)]}{z^2 - z(e^{i\theta}+e^{-i\theta})+1}$$

$$= \frac{z[(z-\cos\theta)+i\sin\theta]}{z^2-2z\cos\theta+1}$$

$$Z_T[\cos n\theta + i\sin n\theta] = \frac{z[(z-\cos\theta)+i\sin\theta]}{z^2-2z\cos\theta+1}$$

$$Z_T[\cos n\theta] = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$$

$$Z_T[\sin n\theta] = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$$

4m

$$Z_T[k^n \cos n\theta] = \frac{z(z - k\cos\theta)}{z^2 - 2kz\cos\theta + k^2}$$

$$Z_T[k^n \sin n\theta] = \frac{kz\sin\theta}{z^2 - 2kz\cos\theta + k^2}$$

$$Z_T[e^{-an}\cos n\theta] = \frac{z(z - e^{-a}\cos\theta)}{z^2 - 2e^{-a}z\cos\theta + e^{-2a}}$$

$$Z_T[e^{-an}\sin n\theta] = \frac{e^{-a}z\sin\theta}{z^2 - 2e^{-a}z\cos\theta + e^{-2a}}$$

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3. 3m

$$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & |x| > a \end{cases}$$

Solution

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{iux}\, dx$$ 1m

$$= \int_{-1}^{1} 1 \times e^{iux}\, dx = \left(\frac{e^{iux}}{iu}\right)_{x=-1}^{1}$$

$$F(u) = \frac{e^{iu} - e^{-iu}}{iu}$$

Substitute $e^{iu} = \cos u + i\sin u$

& $e^{-iu} = \cos u - i\sin u$

$$F(u) = \frac{2i\sin u}{iu} = \frac{2\sin u}{u}$$

Now $F(u) = \frac{2\sin u}{u}$ 2m

$\therefore$ Applyin inverse fourier transform

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(u)\, e^{-iux}\, du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2\sin u}{u}\, e^{-iux}\, du$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin u}{u}\, e^{-iux}\, dx$$ 2m

Put $x=0$ & 0 is a point of continuity & $f(0)=1$

$$\therefore \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin u}{u}\, du = 1. \quad \left(\text{since } e^{0}=1\right)$$ 1m

$$\Rightarrow \frac{2}{\pi}\int_{0}^{\infty} \frac{\sin u}{u}\, du = 1 \quad \left(\text{since } \frac{\sin u}{u} \text{ is even}\right.$$

$$\Rightarrow \int_{0}^{\infty} \frac{\sin u}{u}\, du = \pi/2$$

$$\boxed{\text{Hence } \int_{0}^{\infty} \frac{\sin x}{x}\, dx = \pi/2}$$ 1m

4.

$$F_S(\alpha) = \frac{1}{\alpha} e^{-a\alpha}$$

$$f(x) = \frac{2}{\pi} \int_0^\infty F_S(\alpha) \sin \alpha x \, d\alpha.$$ 1m

$$f(x) = \frac{2}{\pi} \int_0^\infty \frac{e^{-a\alpha}}{\alpha} \sin \alpha x \, d\alpha$$

$$= \frac{2}{\pi} \phi(x)$$

~~1m~~

$$\phi(x) = \int_0^\infty \frac{e^{-a\alpha}}{\alpha} \sin \alpha x \, d\alpha$$

By Leibnitz rule.

$$\phi'(x) = \int_0^\infty \frac{e^{-a\alpha}}{\alpha} \cos \alpha x . \alpha \, d\alpha.$$

$$= \int_0^\infty e^{-a\alpha} \cos \alpha x . d\alpha$$ 2m

$$\phi'(x) = \left[\frac{e^{-a\alpha}}{a^2+x^2} \left(-a \cos \alpha x + x \sin \alpha x \right) \right]_0^\infty$$

$$\phi'(x) = \frac{1}{a^2+x^2} \left\{ 0 - (-a) \right\} = \frac{a}{a^2+x^2}$$

$$\phi(x) = \int \frac{a}{a^2+x^2} dx + c$$ 2m

$\phi(x) = \tan^{-1}\left(\frac{x}{a}\right) + c.$ Put $x = 0$

1m

$\Rightarrow \phi(0) = \tan^{-1} 0 + c \Rightarrow c = 0 \Rightarrow \phi(x) = \tan^{-1}\left(\frac{x}{a}\right)$

$\therefore \quad f(x) = \frac{2}{\pi} \tan^{-1} (x/a)$ 1m

(Mech-A)

$f(x,y,y') = y'^2 - y^2 + 2y\sec x$

Euler's eq $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ 1m

$(-2y + 2\sec x) - \frac{d}{dx} 2y' = 0$.

$\Rightarrow y'' + y = \sec x \quad \Rightarrow (D^2+1)y = \sec x$.

A eq $m^2 + 1 = 0 \Rightarrow m = \pm i$

$y_c = C_1 \cos x + C_2 \sin x$. 2m

Let $y = A\cos x + B\sin x$ be the sol of ODE where A & B are funcs of x to be determined. If y_1 & y_2 are the solutions of the homogeneous eq $f(D)y = 0$ then W.K.T

$$A = -\int \frac{y_2\, \phi(x)}{W} dx, \quad B = \int \frac{y_1\, \phi(x)}{W} dx.$$ 1m

$\phi(x) = \sec x, \quad W = y_1 y_2' - y_2 y_1'$

$y_1 = \cos x, \quad y_2 = \sin x, \quad y_1' = -\sin x$.

$y_2' = \cos x, \quad W = 1$ 1m

$A = -\int \sin x \sec x\, dx, \quad B = \int \cos x \sec x\, dx$

$A = -\int \tan x\, dx = -\log(\sec x), \quad B = x + k_2$.

$\therefore y = A\cos x + B\sin x$. 1m

$y = [-\log(\sec x) + k_1]\cos x + (x + k_2)\sin x$.

$y = k_1 \cos x + k_2 \sin x - \cos x \log(\sec x) + x\sin x$. 1m

5) The normal eqs for $y = ax^2 + bx + c$

is $\Sigma y = a \Sigma x^2 + b \Sigma x + nc$

$\Sigma xy = a \Sigma x^3 + b \Sigma x^2 + c \Sigma x$ 1m

$\Sigma x^2 y = a \Sigma x^4 + b \Sigma x^3 + c \Sigma x^2 \; (n=5)$

x	y	xy	x^2y	x^2	x^3	x^4
1	10	10	10	1	1	1
2	12	24	48	4	8	16
3	13	39	117	9	27	81
4	16	64	256	16	64	256
5	19	95	475	25	125	625

2m

$\Sigma x = 15$, $\Sigma y = 70$, $\Sigma xy = 232$, $\Sigma x^2 y = 906$,

$\Sigma x^2 = 55$, $\Sigma x^3 = 225$, $\Sigma x^4 = 979$. 1m

$55a + 15b + 5c = 70$

$225a + 55b + 15c = 232$ 1m

$979a + 225b + 55c = 906$.

on solving

$a = 0.2857$, $b = 0.4857$. 1m

$c = 9.4$

$y = 0.29x^2 + 0.49x + 9.4$. 1m

8. 6)

$y = ae^{bx}$.

Take log on b.s

$\log y = \log a + bx \log e$ 1m

$Y = A + bx$.

The normal eqs $\Sigma Y = nA + b\Sigma x$. 1m

$\Sigma xY = A\Sigma x + b\Sigma x^2$

x	y	$Y = \log_e y$	xY	x^2
5	133	4.8903	24.4515	25
6	55	4.0073	24.0438	36
7	23	3.1355	21.9485	49
8	7	1.9459	15.5672	64
9	2	0.6931	6.2379	81
10	2	0.6931	6.9310	100

2m

$\Sigma x = 45$, $\Sigma Y = 15.3652$, $\Sigma xY = 99.1799$

$\Sigma x^2 = 355$. 1m

$6A + 45b = 15.3652$

$45A + 355b = 99.1799$

$\therefore y = 12623 e^{-0.9177x}$ 1m

on solving

$A = 9.4433$, $b = -0.9177$ 1m

$\log_e a = A \Rightarrow a = e^A = e^{9.4433} = 12623.3$

7.

$\bar{x} = \frac{\Sigma x}{n}$, $\bar{y} = \frac{\Sigma y}{n}$ 1m

$\bar{x} = 4$, $\bar{y} = 11$

$X = x - \bar{x}$, $Y = y - \bar{y}$

x	y	X	Y	XY	X^2	Y^2
1	9	−3	−2	6	9	4
2	8	−2	−3	6	4	9
3	10	−1	−1	1	1	1
4	12	0	1	0	0	1
5	11	1	0	0	1	0
6	13	2	2	4	4	4
7	14	3	3	9	9	9

2m

$\Sigma XY = 26$, $\Sigma X^2 = 28$, $\Sigma Y^2 = 28$

The regression lines is in the form

$Y = \frac{\Sigma XY}{\Sigma X^2} \cdot X$, $X = \frac{\Sigma XY}{\Sigma Y^2} \cdot Y$ 1m

$y - 11 = \frac{26}{28}(x - 4)$ $x - 4 = \frac{26}{28}(y - 11)$

$y - 11 = 0.93(x - 4)$ $x = 0.93y - 6.25$

$r = \pm\sqrt{\text{coeff } x \text{ coeff } y} = \pm\sqrt{(0.93)(0.93)} = +0.93$ 3m

8) $\oint_C f\cdot dr = \int_{OA} f\cdot dr + \int_{AB} f\cdot dr + \int_{BP} f\cdot dr$

For IS, civil A, B

Point A is on x axis $\therefore y=0 \quad z=0$

$\therefore f = (x^2 - 27)i \quad \& \quad r = xi \Rightarrow dr = dx\,i$

$\int_{OA} f\cdot dr = \int_0^1 (x^2-27)\,dx = -80/3$

From A to B y varies from 0 to 1 ; $x=1$ & $z=0$

$f = (1-27)i \quad ; \quad r = i + yj \quad dr = dy\,j$

$\int_{AB} f\cdot dr = \int_{y=0}^{1} 0 = 0$

On BP $x=1$; $y=1$ z ~~[illegible]~~ varies from 0 to 1

$f = (1-27)i - 6zj + 8z^2 k$.

$r = i + j + zk$

$dr = dz\,k$.

$\int_{BP} f\cdot dr = \int_{z=0}^{1} 8z^2\,dz = 8/3$

$\therefore \int_C f\cdot dr = -\frac{80}{3} + 0 + \frac{8}{3} = -24$

	Course Outcomes	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Evaluate the real form of the Fourier series for standard periodic and finite waveforms as half-range series which has its applications in finding out the sum of infinite series using Dirichlet's conditions.	3	0	0	0	0	0	0	0	0	0	0	0
CO2:	Apply integral expressions for the forward and inverse Fourier transform to a range of non-periodic waveforms such as rectangular, unit-step, sinusoidal and exponential decay functions and solve second order difference equations using Z transform and inverse Z transform.	3	0	0	0	0	0	0	0	1	0	0	0
CO3:	Solve Brachistochrone, shortest distance, minimal surface area and hanging chain problems and to find the geodesics of known surfaces using Euler-Lagrange method.	3	0	0	0	0	0	0	0		0	0	0
CO4:	Estimate the strength of the relationship between the variables using correlation coefficients and express the relationship in the form if an equation using regression analysis.	3	0	0	0	0	0	0	0	1	0	0	0
CO5:	Evaluate line and surface integrals using Green's, Stoke's and Gauss divergence theorems which have its application in computing the amount of work done, area and volume.	3	0	0	0	0	0	0	0		0	0	0
CO6:	Apply numerical techniques to perform various mathematical tasks such as solving equations, interpolation, integration and curve fitting.	3	0	0	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*