23

LATERAL ENTRY

CMR
INSTITUTE OF
TECHNOLOGY

USN

CMR

Internal Assesment Test - II

Sub:	Engineering Maths-III							Code:	15MAT31
Date:	02 / 11 /2016	Duration:	90 mins	Max Marks:	50	Sem:	3	Branch:	EC-E,CS-D, ME-B

NOTE: First question is compulsory. Answer any six questions from the rest.

Marks | OBE: CO | RBT

1 .The following table gives the variations of a periodic current A over a period T. [8] CO1 L3

t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A(amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a constant part of 0.75 amp in the current A and also obtain the amplitude upto first harmonic.

2 .Obtain the Fourier series expansion for the function $f(x)=|x|\ in(-\pi,\pi)$ .Sketch the graph. [7] CO1 L3

3 .Obtain the half range cosine series for the function $f(x)=(x-1)^2$,in $0\le x\le 1$. [7] CO1 L3

4. Fit a second degree parabola $y=ax^2+bx+c$ for the given data by the method of least squares. [7] CO6 L2

x	1	2	3	4	5
y	10	12	13	16	19

5. Fit an exponential curve of the form $y=ae^{bx}$ by the method of least squares for the following data. [7] CO6 L2

No.of petals	5	6	7	8	9	10
No.of flowers	133	55	23	7	2	2

6. Obtain the lines of regression and hence find the coefficient of correlation for the following data . [7] CO6 L2

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

7 . Fit a straight line of the form $y=ax+b$ by the method of least squares for the following data. [7] CO6 L2

x	50	70	100	120
y	12	15	21	25

8 .Show that if θ is the angle between the lines of regression, then [7] CO6 L3

$$\tan\theta=\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\left\{\frac{1-r^2}{r}\right\}.$$

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Evaluate the real form of the Fourier series for standard periodic and a periodic waveforms which has its applications in finding out the sum of infinite series using Dirichlet's conditions.	3	0	0	0	0	0	0	0	0	0	0	0
CO2:	Apply integral expressions for the forward and inverse Fourier transform to a range of non-periodic waveforms such as rectangular, unit-step, sinusoidal and exponential decay functions and to solve second order difference equations using Z transform and inverse Z transform	3	0	0	0	0	0	0	0	1	0	0	0
CO3:	Apply numerical techniques to perform various mathematical task such as solving equations, interpolation, integration and curve fitting	3	0	0	0	0	0	0	0	1	0	0	0
CO4:	Solve Brachistochrone, shortest distance, minimal surface area and hanging chain problems and to find the geodesics of known surfaces using Euler-Lagrange method.	3	0	0	0	0	0	0	0	0	0	0	0
CO5:	Evaluate line and surface integrals using Green's, Stoke's and Gauss divergence theorems which have its application in computing the amount of work done, area and volume.	3	0	0	0	0	0	0	0	0	0	0	0
CO6:	Estimate the strength of the relationship between the variables using correlation coefficients and to express the relationship in the form if an equation using regression.	3	0	0	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

Internal - 2. November 2016

Solution (Lateral entry)

$2l = T \Rightarrow l = T/2$

$\theta = \frac{\pi t}{T/2} = \frac{2\pi t}{T}$ 1m

$\theta°$	y	$y \cos\theta$	$y \sin\theta$
0	1.98	1.98	0
60	1.3	0.65	1.1258
120	1.05	−0.525	0.9093
180	1.3	−1.3	0
240	−0.88	0.44	0.76208
300	−0.25	−0.125	0.2165

3m

$\Sigma y = 4.5$, $\Sigma y\cos\theta = 1.12$, $\Sigma y \sin\theta = 3.0136$

1m

$a_0 = \frac{2}{N}\Sigma y = \frac{1}{3}(4.5) \Rightarrow \frac{a_0}{2} = 0.75$

$a_1 = \frac{2}{N}\Sigma y \cos\theta = \frac{1.12}{3} = 0.3733$

$b_1 = \frac{2}{N}\Sigma y \sin\theta = \frac{3.0136}{3} = 1.0045$ 2m

Constant part = 0.75 amp

Amplitude of 1st harmonic = 1.07168 1m

$\sqrt{a_1^2 + b_1^2}$

2. $f(x) = |x|$ in $(-\pi, \pi)$

$f(-x) = |-x| = |x|$

∴ The given func is even. 1m

$$a_0 = \frac{2}{\pi}\int_0^\pi f(x)dx = \frac{2}{\pi}\int_0^\pi x\,dx = \frac{2}{\pi}\left(\frac{x^2}{2}\right)$$

$$= \pi.$$

$$\frac{a_0}{2} = \pi/2$$

2m

$$a_n = \frac{2}{\pi}\int_0^\pi x\cos nx\,dx.$$

$$a_n = \frac{2}{\pi}\left[x\frac{\sin nx}{n} - 1.\left(-\frac{\cos nx}{n^2}\right)\right]_0^\pi$$

$$= \frac{2}{\pi n^2}\left[\cos nx\right]_0^\pi = \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

$$= -\frac{2}{\pi n^2}\left\{1 - (-1)^n\right\}$$

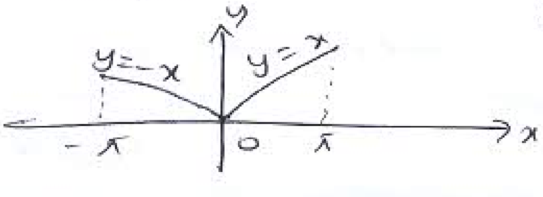
2m

$$a_n = \left\{\begin{matrix} -\dfrac{4}{\pi n^2} & \text{when } n \text{ is odd} \\ 0 & \text{when } n \text{ is even} \end{matrix}\right\}$$

$$f(x) = \frac{\pi}{2} - \sum_{n=1,3,5 --}^{\infty} \frac{-4}{\pi n^2}\cos nx.$$

1m

<u>Graph</u> $|x| = \left\{\begin{matrix} +x, & x > 0 \\ -x, & x < 0 \end{matrix}\right\}$



1m

3.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} \quad , 0 \le x \le 1$$

$l = 1$ 1m

$$a_0 = \frac{2}{l} \int_0^l f(x)\,dx$$

$$= 2 \int_0^1 (x-1)^2\,dx = 2 . \left[\frac{(x-1)^3}{3} \right]_0^1$$

$$= 2/3$$

$$\frac{a_0}{2} = 1/3 .$$ 2m

$$a_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l}\,dx$$ 1m

$$= 2 \int_0^1 (x-1)^2 \cos n\pi x\,dx$$

$$= 2 \left[(x-1)^2 \left(\frac{\sin n\pi x}{n\pi} \right) - 2(x-1)\left(- \frac{\cos n\pi x}{n^2\pi^2} \right) + 2\left(- \frac{\sin n\pi x}{n^3\pi^3} \right) \right]_0^1$$

$$= 2 \left[2(x-1) \frac{\cos n\pi x}{n^2\pi^2} \right]_0^1 \quad = \frac{4}{n^2\pi^2} (0 - (-1))$$

$$= \frac{4}{n^2\pi^2}$$ 2m

$$f(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2} \cos n\pi x$$ 1m

4.

The normal eqs for $y = ax^2 + bx + c$

is $\Sigma y = a\Sigma x^2 + b\Sigma x + nc$

$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$ 1m

$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$ $(n=5)$

x	y	xy	x^2y	x^2	x^3	x^4
1	10	10	10	1	1	1
2	12	24	48	4	8	16
3	13	39	117	9	27	81
4	16	64	256	16	64	256
5	19	95	475	25	125	625

2m

$\Sigma x = 15$, $\Sigma y = 70$, $\Sigma xy = 232$, $\Sigma x^2 y = 906$,

$\Sigma x^2 = 55$, $\Sigma x^3 = 225$, $\Sigma x^4 = 979$. 1m

$55a + 15b + 5c = 70$

$225a + 55b + 15c = 232$ 1m

$979a + 225b + 55c = 906$

on solving

$a = 0.2857$, $b = 0.4857$. 1m

$c = 9.4$

$y = 0.29x^2 + 0.49x + 9.4$. 1m

5. $y = ae^{bx}$.

Take log on b.s

$\log y = \log a + bx \log e$ 1m

$Y = A + bx$.

The normal eqs $\Sigma Y = nA + b\Sigma x$.

$\Sigma xY = A\Sigma x + b\Sigma x^2$ 1m

x	y	$Y = \log_e y$	xY	x^2
5	133	4.8903	24.4515	25
6	55	4.0073	24.0438	36
7	23	3.1355	21.9485	49
8	7	1.9459	15.5672	64
9	2	0.6931	6.2379	81
10	2	0.6931	6.9310	100

2m

$\Sigma x = 45$, $\Sigma Y = 15.3652$, $\Sigma xY = 99.1799$

$\Sigma x^2 = 355$. 1m

$6A + 45b = 15.3652$

$45A + 355b = 99.1799$

$\therefore y = 12623e^{-0.9177x}$ 1m

on solving

$A = 9.4433$, $b = -0.9177$ 1m

$\log_e a = A \Rightarrow a = e^A = e^{9.4433} = 12623.3$

6.

$\bar{x} = \frac{\sum x}{n}$, $\bar{y} = \frac{\sum y}{n}$ 1m

$\bar{x} = 4$, $\bar{y} = 11$

$X = x - \bar{x}$, $Y = y - \bar{y}$

x	y	X	Y	XY	X^2	Y^2
1	9	−3	−2	6	9	4
2	8	−2	−3	6	4	9
3	10	−1	−1	1	1	1
4	12	0	1	0	0	1
5	11	1	0	0	1	0
6	13	2	2	4	4	4
7	14	3	3	9	9	9

2m

$\sum XY = 26$, $\sum X^2 = 28$, $\sum Y^2 = 28$

The regression lines is in the form

$Y = \frac{\sum XY}{\sum X^2} \cdot X$, $X = \frac{\sum XY}{\sum Y^2} \cdot Y$ 1m

$y - 11 = \frac{26}{28}(x - 4)$ $x - 4 = \frac{26}{28}(y - 11)$

$y - 11 = 0.93(x - 4)$ $x = 0.93y - 6.25$

$r = \pm\sqrt{\text{coeff } x \text{ coeff } y} = \pm\sqrt{(0.93)(0.93)} = +0.93$ 3m

$r = \sqrt{(\text{coeff of } x)(\text{coeff of } y)}$ 1m

$= \sqrt{(0.93)(0.93)} = +0.93$

$r = +0.93$ 1m

$y = ax+b$

The normal eqs are.

$\Sigma y = a\Sigma x + nb$ 1m

$\Sigma xy = a\Sigma x^2 + b\Sigma x$.

x	y	xy	x^2
50	12	600	2500
70	15	1050	4900
100	21	2100	10,000
120	25	3000	14,400

2m

$\Sigma x = 340$, $\Sigma y = 73$, $\Sigma xy = 6750$,
$\Sigma x^2 = 31800$ 1m

The normal eqs

$340a + 4b = 73$

$31800a + 340b = 6750$ 1m

on solving $a = 0.1879$, $b = 2.2758$ 1m

$y = 0.19x + 2.28$ 1m

8. W.K.T if θ is acute, the angle b/w the lines $y = m_1 x + c_1$ & $y = m_2 x + c_2$ is

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$ 1m

The lines of regression are

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x}) \quad - ①$$

1m

$$\& \; x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y}) \quad - ②$$

rewrite ② as $\frac{\sigma_y}{r\sigma_x}(x - \bar{x}) = (y - \bar{y}) - ③$ 1m

Slopes of ① & ③ are

$m_1 = r\frac{\sigma_y}{\sigma_x}$ & $m_2 = \frac{\sigma_y}{r\sigma_x}$ 1m

$$\therefore \tan\theta = \frac{\frac{\sigma_y}{r\sigma_x} - \frac{r\sigma_y}{\sigma_x}}{1 + r\frac{\sigma_y}{\sigma_x} \cdot \frac{\sigma_y}{r\sigma_x}} = \frac{\frac{\sigma_y}{\sigma_x}\left[\frac{1-r^2}{r}\right]}{\frac{\sigma_x^2 + \sigma_y^2}{\sigma_x^2}}$$

2m

$$\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left[\frac{1-r^2}{r}\right]$$ 1m