

Improvement Test

Sub:	Engineering Maths-III	Code:	15MAT31
Date:	18 / 11 / 2016	Duration:	90 mins
		Max Marks:	50
		Sem:	3
		Branch:	ME-B,EC-E,CS-D

NOTE: First question is compulsory. Answer any six questions from the rest.

	Marks	OBE															
		CO	RBT														
1. Evaluate $\int_0^{\frac{\pi}{2}} \cos x \, dx$ by Simpson's one third rule by taking eleven ordinates. Compare the value with the theoretical value.	[8]	CO3	L3														
2. Apply Lagrange's formula inversely to find a root of the equation $f(x) = 0$ given that $f(30) = -30, f(34) = -13, f(38) = 3, f(42) = 18$ .	[7]	CO3	L3														
3. Find $y(1.4)$ by Newton's forward interpolation formula from the given data	[7]	CO3	L3														
<table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr><td style="padding: 5px;">x</td><td style="padding: 5px;">1</td><td style="padding: 5px;">2</td><td style="padding: 5px;">3</td><td style="padding: 5px;">4</td><td style="padding: 5px;">5</td></tr> <tr><td style="padding: 5px;">y</td><td style="padding: 5px;">10</td><td style="padding: 5px;">26</td><td style="padding: 5px;">58</td><td style="padding: 5px;">112</td><td style="padding: 5px;">194</td></tr> </table>	x	1	2	3	4	5	y	10	26	58	112	194					
x	1	2	3	4	5												
y	10	26	58	112	194												
4. Find the correlation coefficient and the equation of lines of regression from the given data	[7]	CO6	L3														
<table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr><td style="padding: 5px;">x</td><td style="padding: 5px;">1</td><td style="padding: 5px;">2</td><td style="padding: 5px;">3</td><td style="padding: 5px;">4</td><td style="padding: 5px;">5</td></tr> <tr><td style="padding: 5px;">y</td><td style="padding: 5px;">2</td><td style="padding: 5px;">5</td><td style="padding: 5px;">3</td><td style="padding: 5px;">8</td><td style="padding: 5px;">7</td></tr> </table>	x	1	2	3	4	5	y	2	5	3	8	7					
x	1	2	3	4	5												
y	2	5	3	8	7												
5. If $x$ and $y$ are random variables with standard deviations $\sigma_x$ and $\sigma_y$ , it was found that random variables $(x+y), (x-y), (2x+y)$ respectively have variance 15, 11, 29. Compute standard deviations of $x, y$ and also the coefficient of correlation.	[7]	CO6	L3														
6. Fit a straight line of the form $y = ax + b$ for the following data	[7]	CO3	L3														
<table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr><td style="padding: 5px;">x</td><td style="padding: 5px;">5</td><td style="padding: 5px;">10</td><td style="padding: 5px;">15</td><td style="padding: 5px;">20</td><td style="padding: 5px;">25</td></tr> <tr><td style="padding: 5px;">y</td><td style="padding: 5px;">16</td><td style="padding: 5px;">19</td><td style="padding: 5px;">23</td><td style="padding: 5px;">26</td><td style="padding: 5px;">30</td></tr> </table>	x	5	10	15	20	25	y	16	19	23	26	30					
x	5	10	15	20	25												
y	16	19	23	26	30												
7. Find the negative root of the equation $x^3 - 4x + 9 = 0$ by Regula-Falsi method. Carry out two iterations.	[7]	CO3	L3														
8. Obtain the constant term and the coefficients of first cosine and sine terms in the Fourier expansion of $y$ from the table	[7]	CO1	L3														
<table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr><td style="padding: 5px;">x</td><td style="padding: 5px;">0</td><td style="padding: 5px;">1</td><td style="padding: 5px;">2</td><td style="padding: 5px;">3</td><td style="padding: 5px;">4</td><td style="padding: 5px;">5</td></tr> <tr><td style="padding: 5px;">y</td><td style="padding: 5px;">9</td><td style="padding: 5px;">18</td><td style="padding: 5px;">24</td><td style="padding: 5px;">28</td><td style="padding: 5px;">26</td><td style="padding: 5px;">20</td></tr> </table>	x	0	1	2	3	4	5	y	9	18	24	28	26	20			
x	0	1	2	3	4	5											
y	9	18	24	28	26	20											

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Evaluate the real form of the Fourier series for standard periodic and a periodic waveforms which has its applications in finding out the sum of infinite series using Dirichlet's conditions.	3	0	0	0	0	0	0	0	0	0	0	0
CO2:	Apply integral expressions for the forward and inverse Fourier transform to a range of non-periodic waveforms such as rectangular, unit-step, sinusoidal and exponential decay functions and to solve second order difference equations using Z transform and inverse Z transform	3	0	0	0	0	0	0	0	1	0	0	0
CO3:	Apply numerical techniques to perform various mathematical task such as solving equations, interpolation, integration and curve fitting	3	0	0	0	0	0	0	0	1	0	0	0
CO4:	Solve Brachistochrone, shortest distance, minimal surface area and hanging chain problems and to find the geodesics of known surfaces using Euler-Lagrange method.	3	0	0	0	0	0	0	0	0	0	0	0
CO5:	Evaluate line and surface integrals using Green's, Stoke's and Gauss divergence theorems which have its application in computing the amount of work done, area and volume.	3	0	0	0	0	0	0	0	0	0	0	0
CO6:	Estimate the strength of the relationship between the variables using correlation coefficients and to express the relationship in the form if an equation using regression.	3	0	0	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - Engineering knowledge; PO2 - Problem analysis; PO3 - Design/development of solutions; PO4 - Conduct investigations of complex problems; PO5 - Modern tool usage; PO6 - The Engineer and society; PO7- Environment and sustainability; PO8 - Ethics; PO9 - Individual and team work; PO10 - Communication; PO11 - Project management and finance; PO12 - Life-long learning



# Improvement test.

Maths - 3.

ME - B.  
(Regular)  
Cy dip  
ECE, CSD

1.

$$I = \int_0^{\pi/2} \cos x \, dx.$$

$$h = \frac{\pi}{20} = 9^\circ, n = 10 \quad - 1m$$

$x^\circ$	$0^\circ$	9	18	27	36	45	54	63	72	81	90
$y = \cos x$	1	0.9877	0.9511	0.8910	0.8090	0.7071	0.5878	0.454	0.3090	0.1564	0

$y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6 \quad y_7 \quad y_8 \quad y_9 \quad y_{10}$

By Simpson's  $\frac{1}{3}$ rd rule.

3m

$$I = \frac{h}{3} \left[ (y_0 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8) \right] \quad 1m$$

$$= \frac{1}{3} \cdot \frac{\pi}{20} \left[ (1 + 0) + 4(0.9877 + 0.8910 + 0.7071 + 0.454 + 0.1564) + 2(0.9511 + 0.8090 + 0.5878 + 0.3090) \right]$$

$$I = 1.000000358$$

$$\pi = \frac{22}{7}$$

2m

Theoretical value.

$$\int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2} = \sin \frac{\pi}{2} - 0$$

$$= 1 //$$

1m

5. By data  $\sigma_{x+y}^2 = 15$ ,  $\sigma_{x-y}^2 = 11$

$$\sigma_{2x+y}^2 = 29. \quad - (1) \quad 1m$$

$$\sigma_{ax+by}^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2rab\sigma_x\sigma_y$$

$$\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2 + 2r\sigma_x\sigma_y$$

$$\sigma_{x-y}^2 = \sigma_x^2 + \sigma_y^2 - 2r\sigma_x\sigma_y$$

$$\sigma_{2x+y}^2 = 4\sigma_x^2 + \sigma_y^2 + 4r\sigma_x\sigma_y$$

} 2m

using (1) in these we have

$$\sigma_x^2 + \sigma_y^2 + 2r\sigma_x\sigma_y = 15 \quad - (2)$$

$$4\sigma_x^2 + \sigma_y^2 + 4r\sigma_x\sigma_y = 29 \quad - (4)$$

$$(2) + (3) \quad 2\sigma_x^2 + 2\sigma_y^2 = 26$$

$$\sigma_x^2 + \sigma_y^2 = 13 \quad - (5)$$

$$2 \times (5) + (4) \Rightarrow 6\sigma_x^2 + 3\sigma_y^2 = 51$$

$$2\sigma_x^2 + \sigma_y^2 = 17 \quad - (6)$$

2m

Solving (5) & (6)

$$\sigma_x^2 = 4, \sigma_y^2 = 9.$$

$$\sigma_x = 2, \sigma_y = 3$$

1m

using  $\sigma_x, \sigma_y$  in (2)

$$4 + 9 + 12r = 15, \quad r = \frac{1}{6} = 0.17 \quad 1m$$

(2)

2)

$$f(30) = -30 \quad f(34) = -13, \quad f(38) = 3, \quad f(42) = 18$$

$$x_0 = 30, \quad x_1 = 34, \quad x_2 = 38, \quad x_3 = 42, \quad y_0 = -30, \quad y_1 = -13, \quad y_2 = 3, \quad y_3 = 18$$

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)x_0}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} + \frac{(y-y_0)(y-y_2)(y-y_3)x_1}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} \quad 1m$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)x_2}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} + \frac{(y-y_0)(y-y_1)(y-y_2)x_3}{(y_3-y_0)(y_3-y_1)(y_3-y_2)} \quad 2m$$

$$x(0) = \frac{(13)(-3)(-18)30}{(-17)(-33)(-48)} + \frac{30(-3)(-18)(34)}{(17)(-16)(-31)} + \frac{30(13)(-18)(38)}{(33)(16)(-15)}$$

$$+ \frac{(30)(13)(-3)42}{(48)(31)(15)} \quad 2m$$

$$= -0.7821 + 6.5322 + 33.6818 - 2.2016 \quad 1m$$

$$x(0) = 37.2303 \quad 1m$$

3)

Find $y(1.4)$		$x$	1	2	3	4	5
	$y$		10	26	58	112	194

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
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1 10

16

16

2 26

32

22

6

3 58

54

28

6

4 112

82

5 194

2m

$$y_x = y_0 + r \Delta y_0 + \frac{r(r-1)}{2!} \Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!} \Delta^3 y_0 + \dots$$

$$r = \frac{x-x_0}{h} = \frac{1.4-1}{1} = 0.4 \quad 1m$$

$$\Delta y_0 = 16, \quad \Delta^2 y_0 = 16, \quad \Delta^3 y_0 = 6, \quad \Delta^4 y_0 = 0$$

$$y(1.4) = f(1.4) = 10 + (0.4)16 + \frac{(0.4)(0.4-1)}{2} (16)$$

$$+ \frac{(0.4)(0.4-1)(0.4-2)}{6} (6) \quad (6)$$

$$y(1.4) = 14.864 \quad 1m$$

4)

Correlation coefficient.

x	y	z = x - y	x <sup>2</sup>	y <sup>2</sup>	z <sup>2</sup>
1	2	-1	1	4	1
2	5	-3	4	25	9
3	3	0	9	9	0
4	8	-4	16	64	16
5	7	-2	25	49	4
<u>Σx = 15</u>	<u>Σy = 25</u>	<u>Σz = -10</u>	<u>Σx<sup>2</sup> = 55</u>	<u>Σy<sup>2</sup> = 151</u>	<u>Σz<sup>2</sup> = 30</u>

2m



(3)

$$\sigma_x^2 = \frac{\sum x^2}{n} - (\bar{x})^2 = \frac{55}{5} - (3)^2 = 2$$

$$\sigma_y^2 = \frac{\sum y^2}{n} - (\bar{y})^2 = \frac{151}{5} - (5)^2 = 5.2$$

1 m

$$\sigma_z^2 = \frac{\sum z^2}{n} - (\bar{z})^2 = \frac{30}{5} - (-2)^2 = 2$$

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_z^2}{2\sigma_x\sigma_y} = \frac{2 + 5.2 - 2}{2\sqrt{2}\sqrt{5.2}} \approx 0.81 \quad 1m$$

$$(y - \bar{y}) = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$y - 5 = (0.81) \frac{\sqrt{5.2}}{\sqrt{2}} (x - 3)$$

$$y = 1.306x + 1.082$$

2 m

$$x (\bar{x} - \bar{x}) = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$x - 3 = (0.81) \frac{\sqrt{2}}{\sqrt{5.2}} (y - 5)$$

$$x = 0.502y + 0.49$$

1 m

5) Find the equation of best fitting st line.

x	y	xy	x <sup>2</sup>
5	16	80	25
10	19	190	100
15	23	345	225
20	26	520	400
25	30	750	625
<u>Σx=75</u>	<u>Σy=114</u>	<u>Σxy=1885</u>	<u>Σx<sup>2</sup>=1375</u>

$$75a + 5b = 114$$

$$1375a + 75b = 1885 \quad 1m$$

$$a = 0.7, \quad b = 12.3 \quad 1m$$

$$y = 0.7x + 12.3 \quad \text{is the equation} \quad 1m$$

The normal eqns  
 $\Sigma y = a \Sigma x + nb$   
 $\Sigma xy = a \Sigma x^2 + b \Sigma x$  1m

7)

$$x^3 - 4x + 9$$

$$f(x) = x^3 - 4x + 9, \quad f(-2) = 9 > 0, \quad f(-3) = 6 < 0 \quad 1m$$

∴ Negative root lies between (-3, -2) 1m

neighbourhood of -3

$$f(-2.8) = -1.752, \quad f(-2.7) = 0.117$$

∴ I Iteration

$$a = -2.8$$

$$b = -2.7$$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = -2.7063 \quad 2m$$

II iteration

$$f(-2.7063) = 0.0041 > 0$$

$$a = -2.8, \quad b = -2.7063$$

$$x_2 = -2.7065 \quad 2m$$

∴  $x = -2.7065$  is the root correct to 3 decimal places 1m



8.	$x$	$y$	$\theta^\circ$	$\cos \theta$	$y \cos \theta$	$\sin \theta$	$y \sin \theta$
	0	9	0	1	9	0	0
	1	18	60	0.5	9	0.866	15.588
	2	24	120	-0.5	-12	0.866	20.784
	3	28	180	-1	-28	0	0
	4	26	240	-0.5	-13	-0.866	-25.516
	5	20	300	0.5	10	-0.866	-17.32
							2m

$$y = f(x) = \frac{a_0}{2} + a_1 \cos \theta + b_1 \sin \theta \quad 1m$$

$$\sum y = 125 \quad a_0 = \frac{2}{N} \sum y = \frac{2}{3} (125) = 83.33$$

$$\sum y \cos \theta = -25 \quad a_1 = \frac{2}{N} \sum y \cos \theta = \frac{2}{3} (-25) = -16.67$$

$$\sum y \sin \theta = -3.464 \quad b_1 = \frac{2}{N} \sum y \sin \theta = \frac{2}{3} (-3.464) = -2.31$$

$$\text{Constant term} = \frac{a_0}{2} = 41.67$$

$$\text{coeff of first cosine term} = a_1 = -16.67$$

$$\text{coeff of first sine term} = b_1 = -2.31$$

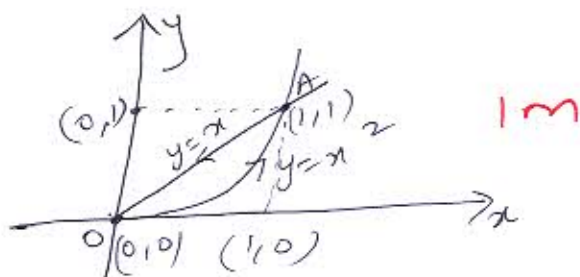
1m

8. By Green's theorem

$$\int_C M dx + N dy = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy \quad 1m$$

The pts of intersection are  $(0,0), (1,1)$   
 $y=x, y=x^2$

$$\int_C (xy + y^2) dx + x^2 dy = \int_{OA} (xy + y^2) dx + \int_{AO} (xy + y^2) dx + x^2 dy$$



$I_1 + I_2$

$$I_1 = \int_{x=0}^1 (x \cdot x^2 + x^4) dx + 2x^3 dx$$

$$I_1 \Rightarrow y = x^2, dy = 2x dx, x \rightarrow 0 \text{ to } 1$$

$$= \int_0^1 (3x^3 + x^4) dx = 3 \left[ \frac{x^4}{4} \right]_0^1 + \left[ \frac{x^5}{5} \right]_0^1 = \frac{19}{20}$$

Along  $I_2$   $y=x, dy=dx, x \rightarrow 1 \text{ to } 0$

$$I_2 = \int_{x=0}^1 (x \cdot x + x^2) dx = \int_1^0 3x^2 dx = -1$$

$$\therefore \int_C (xy + y^2) dx + x^2 dy = \frac{19}{20} - 1 = -\frac{1}{20} \quad 2m$$

$$M = xy + y^2, N = x^2$$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = x - 2y \quad 1m$$

$$= \iint_R (x - 2y) dx dy$$

$$\therefore \text{Theorem is verified} = \int_{x=0}^1 \int_{y=x^2}^x (x - 2y) dy dx = \int_{x=0}^1 (x^2 - x^3) dx = -\frac{1}{20} \quad 2m$$