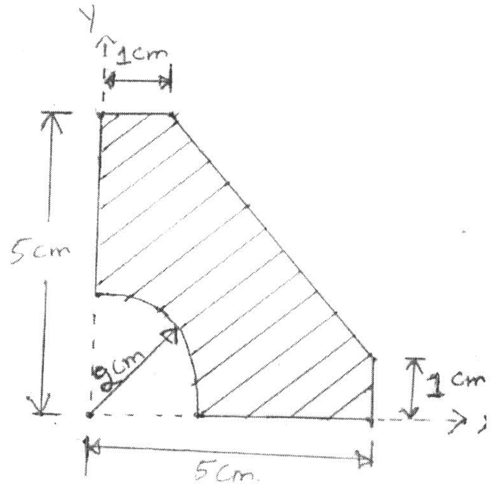
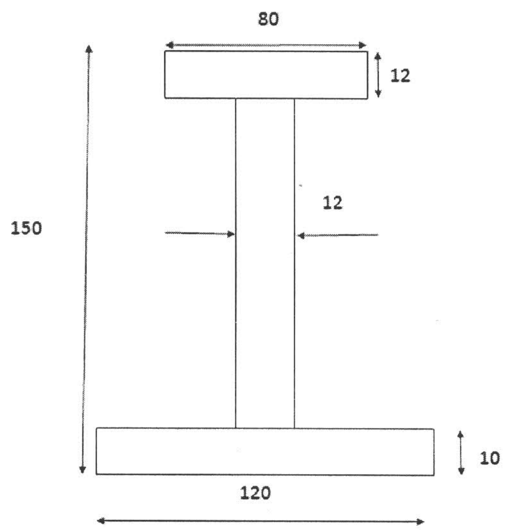
CMR INSTITUTE OF TECHNOLOGY	Roll NO				

Improvementent Test

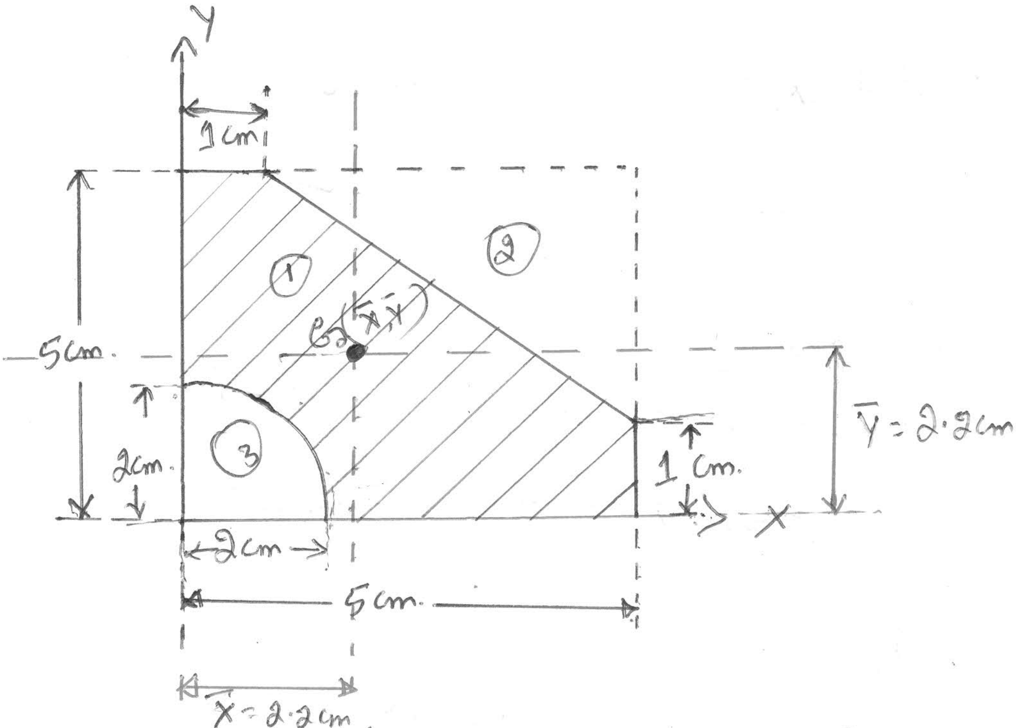
Sub:	Elements of Civil Engineering and Engineering Mechanics						CO104. de:	15CIV23
Date:		Duration:	90 mins	Max Marks:	50	Sem: II	Branch:	All

Note: Answer any five full questions from Q1 to Q7

		Marks	OBE CO	OBE RBT
1(a)	Determine the moment of inertia of the shaded area shown in Fig. 1a about given X and Y axis.  Fig.1a	[10]	CO104.5	L3
2 (a)	Determine polar moment of inertia and radius of gyration about centroidial axis of I section shown in Fig. 2a (All dimension are in **mm**)  Fig. 2a	[10]	CO104.5	L3
3 (a)	Derive an expression for moment of inertia of a triangle about horizontal centroidial axis by method of integration.	[08]	CO104.5	L2

(b)	State parallel axis theorem.	[02]	CO104.5	L1
4 (a)	Define (i) Velocity (ii) Displacement and (iii) Acceleration	[03]	CO104.6	L1
(b)	A car starts from rest and travels on a straight road with a constant acceleration of $1.2 m/s^2$. After some time a scooter passes by it travelling in the opposite direction with a uniform velocity of 36kmph. The scooter reaches the starting position of the car 30 sec after car had left from there. Determine when and where two vehicles passed each other.	[07]	CO104.6	L3
5(a)	Define the following (i) Rectilinear motion (ii) Curvilinear motion (iii) Motion under gravity	[03]	CO104.6	L1
(b)	The velocity of a particle along a straight path is defined by a relation $v = 6t-3t^2$ m/s, where t is in seconds. Knowing that x=0 when t=0. Determine (i) Determine the particle's deceleration and position when t=3 sec. (ii) The distance travelled during these 3 sec.	[07]	CO104.6	L3
6 (a)	With the help of neat sketch explain the following terms angle of projection, time of flight, horizontal range and maximum height attained by the particle.	[04]	CO104.6	L1
(b)	A projectile is fired from the top of a cliff 150m height with an initial velocity of 180m/sec at an upward angle of 30^0 to horizontal. Neglecting air resistance determine, the horizontal distance from the gun point to the point where the projectile strikes the ground.	[06]	CO104.6	L3
7 (a)	Derive the equation of path of the projectile.	[04]	CO104.6	L2
(b)	A ball is projected upwards from the top of a tower 25m high and strikes the ground after 12secs at a point 300m from the foot of the tower. Determine the velocity of projection and also the maximum height attained by the ball above the ground.	[06]	CO104.6	L3

1.(a)



Soln:-

To locate Centroid. [Not required to locate for current question, but should be located if M.I is to be determined about centroidal axes]

Fig.	Area (A) (cm^2)	x_i	y_i	Ax_i	Ay_i
① Square	$=5\times5=25\ cm^2$	$=2.5$	$=2.5$	62.5	62.5
② Triangle	$=-\frac{1}{2}\times4\times4$ $=-8\ cm^2$	$5-\frac{1}{3}(4)$ $=3.67$	$5-\frac{1}{3}(4)$ $=3.67$	-29.36	-29.36
3) Quarter Circle	$=-\frac{\pi\times2^2}{4}$ $=-3.14\ cm^2$	$\frac{4\times2}{3\pi}$ $=0.848$	$=\frac{4\times2}{3\pi}$ $=0.848$	-2.66	-2.66

$\Sigma A = 13.86\ cm^2$ $\boxed{\Sigma A x_i = 30.48}$ $\boxed{\Sigma A y_i = 30.48}$ cm^3.

$$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma A} = \frac{30.48}{13.86} = 2.199\ cm \simeq 2.2\ cm$$

$$\bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A} = \frac{30.48}{13.86} = 2.199\ cm \simeq 2.2\ cm.$$

Solution for current Problem starts here:-

Moment of inertia of each figure about their Centroidal axes.

① Square: $I_{G x_1} = \frac{5 \times 5^3}{12} = 52.08\,cm^4$

$I_{G y_1} = \frac{5 \times 5^3}{12} = 52.08\,cm^4.$

② Triangle

$I_{G x_2} = \frac{4 \times 4^3}{36} = 7.11\,cm^4$

$I_{G y_2} = \frac{4 \times 4^3}{36} = 7.11\,cm^4$

③ Quarter Circle

$I_{G x_3} = 0.055 \times 2^4 = 0.88\,cm^4$

$I_{G y_3} = 0.055 \times 2^4 = 0.88\,cm^4$

$$I_{xx} = 52.08 + 5 \times 5 \left(\frac{5}{2}\right)^2 - 7.11 - \frac{1}{2} \times 4 \times 4 \left(5 - \frac{4}{3}\right)^2 - 0.88 - \frac{\pi \times 2^2}{4} \left(\frac{4 \times 2}{3\pi}\right)^2$$

$$= 52.08 + 156.25 - 7.11 - 107.55 - 0.88 - 2.26$$

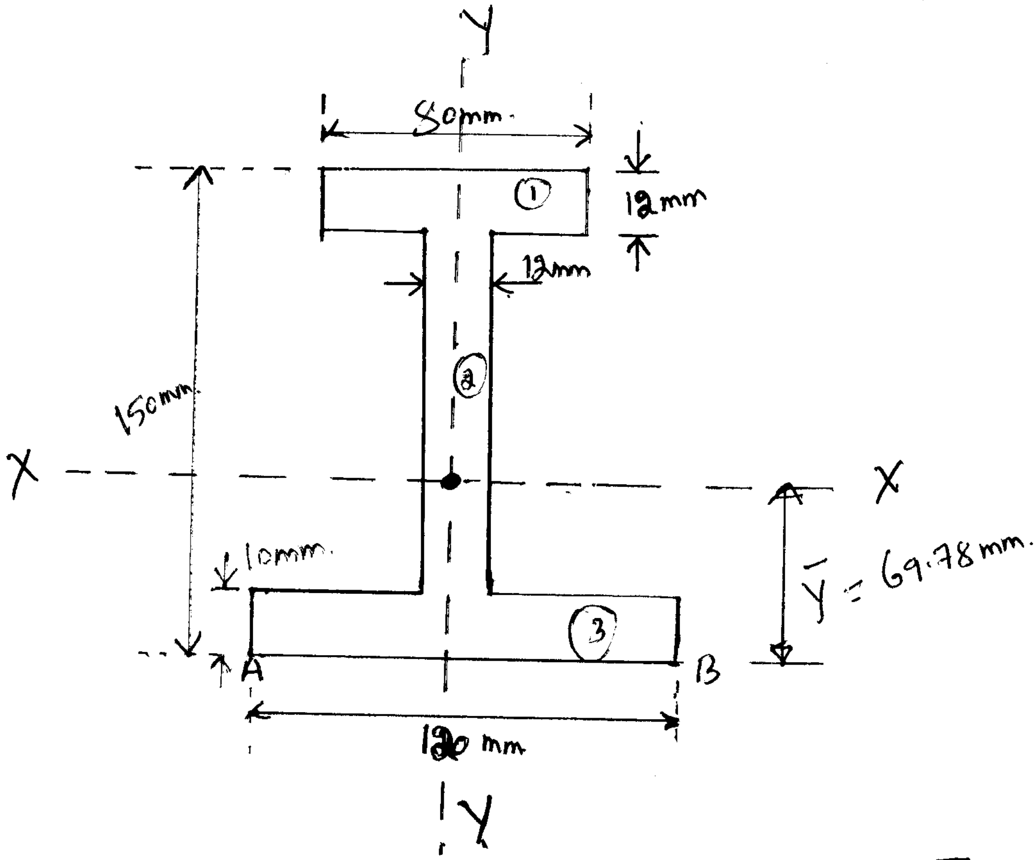
$$\boxed{I_{xx} = 90.53\,cm^4}$$

$I_{yy} = 52.08 +$

As the given sketch is same in both X & Y direction.

$$\boxed{I_{xx} = I_{yy} = 90.53\,cm^4}$$

2.a



Since Section is Symmetric about Y-Y axis $\bar{X} = 0$

To locate Centroid with respect to base line AB

Fig	Area (mm^2)	y_i (mm)	Ay_i
Rect 1	$= 80 \times 12$ $= 960\,mm^2$	$150 - \frac{12}{2}$ $= 144$.	1,38,240
Rect 2	$= 12 \times 128$ $= 1536\,mm^2$	$= 10 + \frac{128}{2}$ $= 74$.	1,13,664
Rect 3	$= 120 \times 10$ $= 1200\,mm^2$	$= \frac{10}{2}$ $= 5$	$= 6,000$

$\Sigma A = 3696\,mm^2$. $\Sigma Ay_i = 2,57,904$

$$\bar{Y} = \frac{\Sigma Ay_i}{\Sigma A} = \frac{2,57,904}{3,696} = 69.78\text{ mm}$$

Moment of inertia of each figure about their Centroidal axes

(1) Rectangle (80×12)

$$I_{Gx_1} = \frac{80 \times 12^3}{12} = 11520\ mm^4$$

$$I_{Gy_1} = \frac{12 \times 80^3}{12} = 512000\ mm^4$$

(2) Rectangle (12×128)

$$I_{Gx_2} = \frac{12 \times 128^3}{12} = \cancel{2097000\ mm^4}\ \cancel{184000\ mm^4}\ 2097152\ mm^4$$

$$I_{Gy_2} = \frac{128 \times 12^3}{12} = \cancel{184000\ mm^4}\ 18432\ mm^4$$

(3) Rectangle (120×10)

$$I_{Gx_3} = \frac{120 \times 10^3}{12} = 10000\ mm^4\ \cancel{144000000\ mm^4}$$

$$I_{Gy_3} = \frac{10 \times 120^3}{12} = 1440000\ mm^4$$

$$I_{xx} = 11520 + 80 \times 12\left(150 - 69.78 - \frac{12}{2}\right)^2 + 2097152 + 12 \times 128\left(10 + \frac{128}{2} - 69.78\right)^2 + 10{,}000 + 120 \times 10\left(69.78 - \frac{10}{2}\right)^2$$

$$= 1.152 \times 10^3 + \cancel{17038.064} + 5288.264 \times 10^3 + 2097.152 \times 10^3 + \cancel{2735}\ 27.353 \times 10^3 + 10 \times 10^3 + 5035.738 \times 10^3$$

$$\boxed{I_{xx} = 12.459 \times 10^6\ mm^4}$$

$$I_{yy} = 51.2 \times 10^4 + 1.843 \times 10^4 + 144 \times 10^4$$

$$\boxed{I_{yy} = 1.97 \times 10^6}\ mm^4$$

Polar moment of Inertia

$$I_{zz} = I_{xx} + I_{yy}$$

$$= 12.46 \times 10^6 + 1.97 \times 10^6$$

$$\boxed{I_{zz} = 14.43 \times 10^6 \ mm^4}$$

~~Real~~ Radius of gyration. $K_{zz} = \sqrt{\dfrac{I_{zz}}{A}}$

$$= \sqrt{\frac{14.43 \times 10^6}{3696}}$$

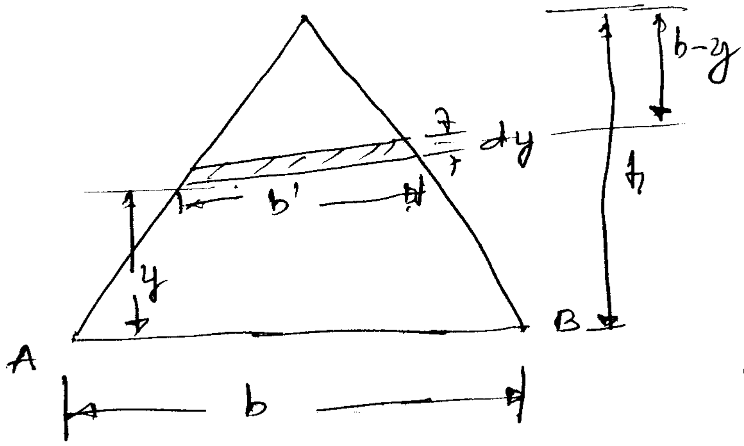
$$\boxed{K_{zz} = 62.48 mm}$$

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{12.46 \times 10^6}{3696}} = \underline{58.06 mm}$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{1.97 \times 10^6}{3696}} = \underline{23.08 mm}$$

3(a) Triangle :-

Consider a ~~turning~~ triangle of base 'b' and height 'h' as shown in figure. choose a horizontal strip of width 'b'' and depth 'dy' at a distance y from the base.



M.I of strip about AB = $b' dy \cdot y^2$

$$\frac{b'}{b} = \frac{h-y}{h}$$

$$b' = b\left(\frac{h-y}{h}\right)$$

$\therefore$ M.I of strip about AB $= b\left(\frac{h-y}{h}\right) \cdot y^2 \cdot dy$

M.I of entire area about AB

$$I_{AB} = \frac{b}{h}\int_0^h (h-y)\, y^2 \cdot dy$$

$$= \frac{b}{h}\int_0^h (hy^2 - y^3) \cdot dy$$

$$= \frac{b}{h}\left[\frac{h \cdot y^3}{3} - \frac{y^4}{4}\right]_0^h$$

$$= \frac{b}{h}\left[\frac{h^4}{3} - \frac{h^4}{4}\right] = \frac{b}{h}\left[\frac{4h^4 - 3h^4}{12}\right]$$

$$I_{AB} = \frac{bh^3}{12}$$

The centroid of triangle is at a height of $\frac{h}{3}$ from the base 'AB'. Using $\parallel^{l}$ axis theorem.

$$I_{AB} = I_G + A.d^2$$

$$A = \frac{1}{2}bh \quad \& \quad d = \frac{h}{3}$$

$$I_G = I_{AB} - A.d^2$$

$$= \frac{bh^3}{12} - \left(\frac{1}{2}bh\right).\left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{3bh^3 - 2bh^3}{36}$$

$$I_G = \frac{bh^3}{36}$$

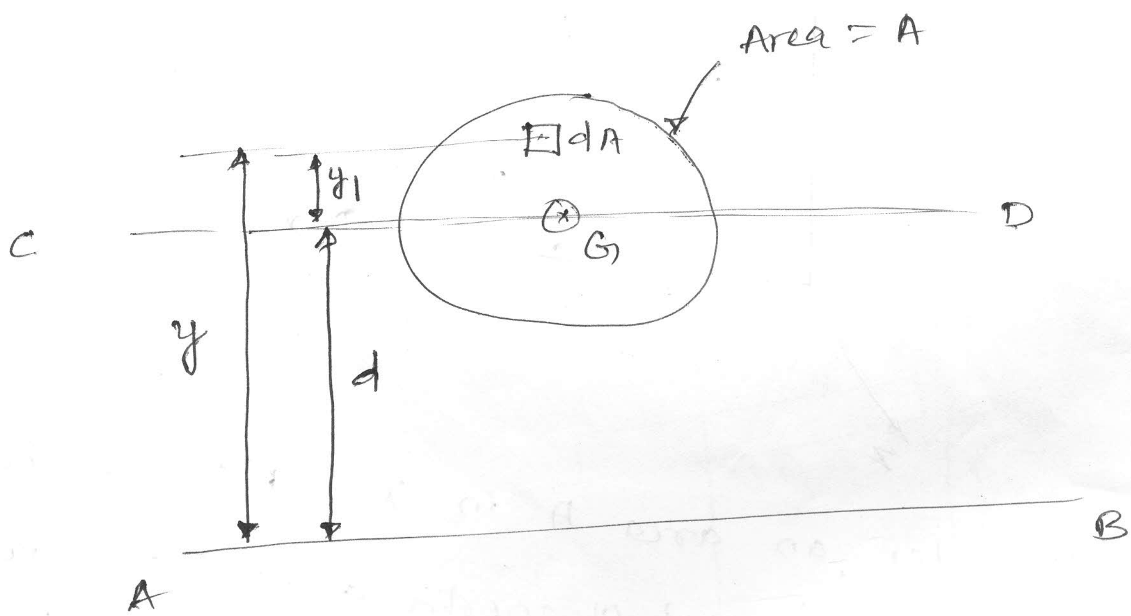
Radius of gyration about centroidal axis

$$K_G = \sqrt{\frac{I_G}{A}} = \sqrt{\frac{bh^3}{36 \times \frac{1}{2} \times b \times h}}$$

$$K_G = \frac{h}{3\sqrt{2}} \quad //$$

Parallel axes theorem:

The moment of inertia of any area about an axis in its plane is the sum of moment of inertia about a parallel axis passing through the centroid of the area (known as centroidal axis) and the product of area and square of the distance between the two parallel axes.



In the above figure

$$I_{AB} = \int y^2 . dA$$

$$y = d + y_1$$

$$I_{AB} = \int (y_1 + d)^2 dA$$

$$= \int (y_1^2 + d^2 + 2 y_1 d) dA$$

$$I_{AB} = \int y_1^2 dA + \int d^2 dA + 2d \int y_1 dA$$

4(a) Define (i) velocity (ii) Displacement (iii) Acceleration.

velocity :- Rate of change of displacement

$$v = \frac{dx}{dt}$$

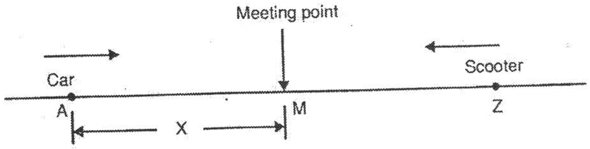
Displacement :- It is a straight line vector joining ~~in~~ initial and final position of particle.

Acceleration :- Rate of change of velocity w.r.t time

$$a = \frac{dv}{dt}.$$

4(b) ~~Ex.9.2~~ A car starts from rest and travels on a straight road with a constant acceleration of 1.2 m/s^2. After some time a scooter passes by it traveling in the opposite direction with a uniform velocity of 36 kmph. The scooter reaches the starting position of the car 30 sec after car had left from there. Determine when and where the two vehicles passed each other.

Solution:



Refer figure. At t = 0, let the car be at position A and the scooter be at position Z. Let the two vehicles pass each other at M which is x metres from A, t sec later.

Motion of car
Uniform acceleration

(A to M)
u = 0
v = – *
s = x
a = 1.2 m/s^2
t = t sec
using $s = ut + \frac{1}{2}at^2$
$x = 0 + \frac{1}{2} \times 1.2 \times t^2$
$x = 0.6\, t^2$ .... (1)

Motion of scooter
Uniform velocity
(Z to A)
v = 36 kmph = 10 m/s
s = ?
t = 30 sec

using $v = \frac{s}{t}$

$10 = \frac{s}{30}$

s = 300 m

∴ the scooter was 300 m away from the car when the car started its motion

(Z to M)
v = 10 m/s
s = (300 – x) metres
t = t sec

using $v = \frac{s}{t}$

$10 = \frac{300 - x}{t}$

x = 300 – 10 t .... (2)

Solving equations (1) and (2)
t = 15.53 sec ..... **Ans.**
x = 144.71 m ..... **Ans.**

5(a) Define the following.

(i) Rectilinear motion.

(ii) curvilinear motion

(iii) Motion under gravity.

Rectilinear motion:- motion of a particle along a straight path is known as rectilinear motion.

e.g. motion of a stone dropped from a building.
motion of a lift. etc.

curvilinear motion :- if a particle or a body is moving along a curved path, the motion is termed as curvilinear motion.

e.g. projetile motion, motion of a car on a ~~[illegible]~~ curve.

Motion under gravity :- The motion of a particle in vertical direction under the influence of constant gravitational acceleration is known as motion under gravity.

5.b] $v = 6t - 3t^2$

$v = \frac{ds}{dt}$

$\frac{ds}{dt} = 6t - 3t^2$

$\int_0^s ds = \int_0^t 6t - 3t^2 \, dt$

$s = 3t^2 - t^3 + C_1$

Given $x=0$, when $t=0$

$s = 3t^2 - t^3 + C_1$

$0 = 0 - 0 + C_1$

$\boxed{C_1 = 0}$

So, $v = 6t - 3t^2$ | $s = 3t^2 - t^3$

$a = 6 - 6t$

(i) when $t = 3$sec

$a = 6 - 6 \times 3$

$a = -12 \text{ m/sec}^2$

$s = 3t^2 - t^3$

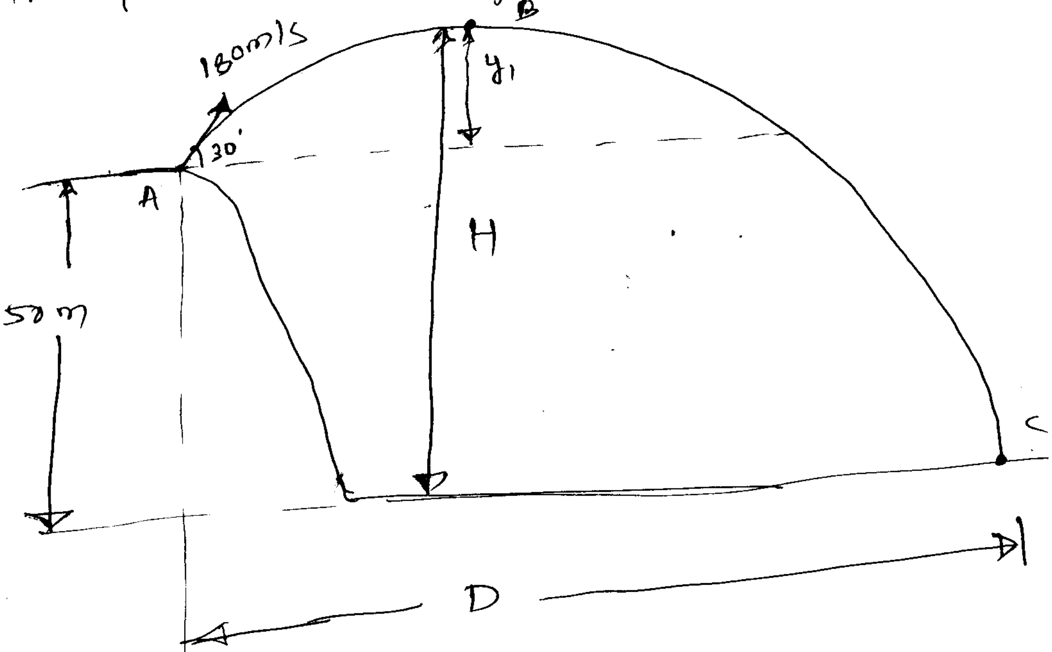
$t = 3$sec

$s = 27 - 27$

$\boxed{s = 0}$

Particle returned to its initial position.

6(b) A projectile is fired from the top of a cliff 150m height with an initial velocity of 180m/s at an upward angle of 30° to horizontal. Neglecting air resistance determine, the horizontal distance from the gun point to the point where projectile strikes the ground.



VM A→B ↑ g=−ve

$v^2 = u^2 - 2gs$

$0 = (180\sin 30)^2 - 2\times 9.81\times y_1$

$y_1 = 412.84\,m$

$\therefore H = 150 + 412.84$

$H = 562.844\,m$

$v = u - gt$

$0 = 180\sin 30 - 9.81\,t_{AB}$

$t_{AB} = 9.17\,s$.

VM B→C ↓ g=+ve

$s = \cancel{ut}^{0} + \frac{1}{2}gt^2$

$562.844 = \frac{1}{2}\times 9.81\times t_{BC}^2$

$t_{BC} = 10.712\,s$

$T = 9.17 + 10.712$
$= 19.88\,s$

HM A→C

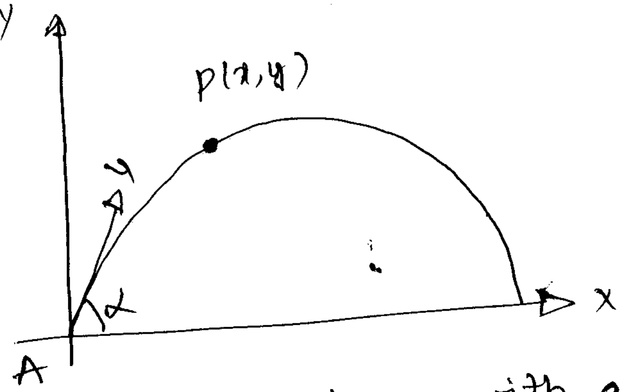
$s = u\times t$

$D = 180\cos 30\times 19.88$

$D = 3098.98\,m$

7(a) Derive the equation of path of the projectile (04)

Ans:-



Consider a projectile thrown with a initial velocity u at angle of α with respect to horizontal. let p(x,y) be the position at any instant of time 't'.

HM A → P

$s = v \times t$

$x = u\cos\alpha \times t$

$t = \dfrac{x}{u\cos\alpha} \longrightarrow (1)$

VM A → P ↑ g = -ve.

$s = ut - \frac{1}{2} g t^2$

$y = (u\sin\alpha).t - \frac{1}{2}.g.t^2 \longrightarrow (2)$

Eqⁿ (1) in Eqⁿ (2).

$y = u\sin\alpha . \dfrac{x}{u\cos\alpha} - \frac{1}{2}.g.\left(\dfrac{x}{u\cos\alpha}\right)^2$

$y = x\tan\alpha - \frac{1}{2}.g.\dfrac{x^2}{u^2\cos^2\alpha}$

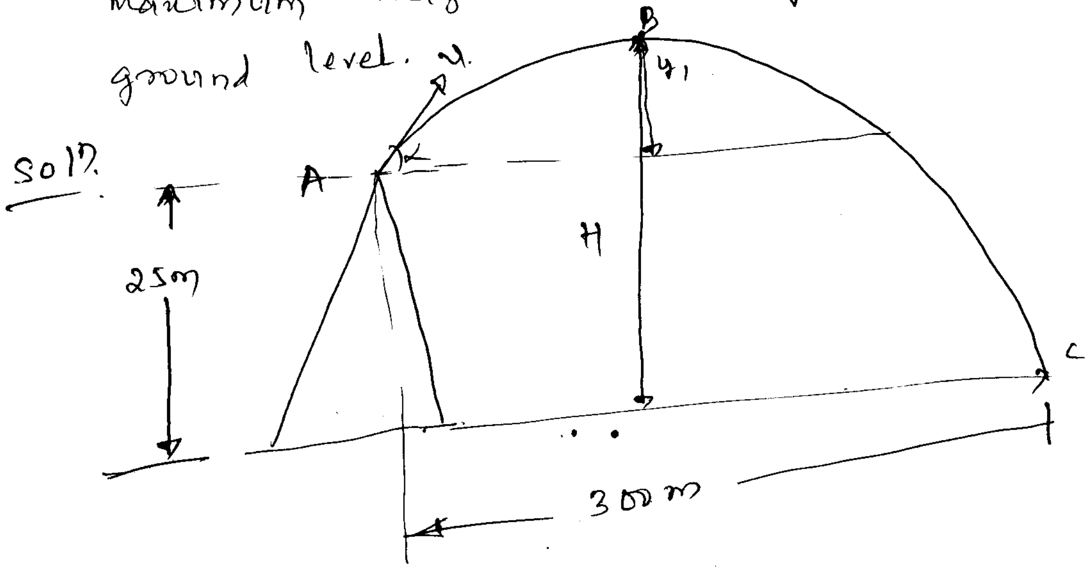
$y = x\tan\alpha - \dfrac{gx^2}{2u^2}\sec^2\alpha$

$y = x\tan\alpha - \dfrac{gx^2}{2u^2}(1+\tan^2\alpha) \longrightarrow (3)$

Eqⁿ (3) is known as path equation of the projectile.

7(b) A ball is projected upwards from the top of a tower 25m high and strikes the ground after 12s at a point 300m from the foot of the tower. Determine the velocity of projection & also the maximum height attained by the ball above the ground level.

Soln.



VM A→C

$S = ut + \frac{1}{2}gt^2$

$-25 = u\sin\alpha \times 12 - \frac{1}{2} \times 9.81 \times (12)^2$

$u\sin\alpha = 56.77 \rightarrow (1)$

HM A→C

$S = v \times t$

$300 = u\cos\alpha \times 12$

$u\cos\alpha = 25 \rightarrow (2)$

$\frac{u\sin\alpha}{u\cos\alpha} = \frac{56.77}{25}$

$\tan\alpha = 2.27$

$\alpha = 66.23'$

$\therefore u = 62.02$ m/s.

$H = 25 + y_1$

VM A→B

$v^2 = u^2 - 2gs$

$0 = (62.02 \sin 66.23)^2 - 2 \times 9.81 \times y_1$

$y_1 = 164.22$ m

$H = 25 + 164.22$

$H = 189.22$ m