(115)

CMR INSTITUTE OF TECHNOLOGY

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CMR

Improvement Test

Sub:	Engineering Maths-III	Code:	15MAT31

Date:	18 / 11 /2016	Duration:	90 mins	Max Marks:	50	Sem:	3	Branch:	IS- B CV -A,B

NOTE: First question is compulsory. Answer any six questions from the rest.

		Marks	OBE CO	OBE RBT
1	Derive Euler's equation in standard form.	[8]	CO4	L3
2	Apply Lagrange's formula inversely to find a root of the equation $f(x)=0$ given that $f(30)=-30, f(34)=-13$, $f(38)=3$, $f(42)=18$.	[7]	CO3	L3
3	Find $y(1.4)$ by Newton's forward interpolation formula from the given data	[7]	CO3	L3

x	1	2	3	4	5
y	10	26	58	112	194

4	Find the correlation coefficient and the equation of lines of regression from the given data.	[7]	CO6	L3

x	1	2	3	4	5
y	2	5	3	8	7

5	Fit a straight line of the form $y = ax + b$ for the following data .	[7]	CO3	L3

x	5	10	15	20	25
y	16	19	23	26	30

6	Find the negative root of the equation $x^3 - 4x + 9 = 0$ by Regula Falsi method Carry out two iterations.	[7]	CO3	L3
7	Evaluate $\int_0^1 \frac{xdx}{1+x^2}$ by Weddle's rule taking seven ordinates and hence find $\log_e 2$	[7]	CO3	L3
8	Find the area between the parabolas $y^2 = 4x$ *and* $x^2 = 4y$ with the help of Green's theorem in a plane.	[7]	CO5	L3

(selected)

	Course Outcomes	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Evaluate the real form of the Fourier series for standard periodic and a periodic waveforms which has its applications in finding out the sum of infinite series using Dirichlet's conditions.	3	0	0	0	0	0	0	0	0	0	0	0
CO2:	Apply integral expressions for the forward and inverse Fourier transform to a range of non-periodic waveforms such as rectangular, unit-step, sinusoidal and exponential decay functions and to solve second order difference equations using Z transform and inverse Z transform	3	0	0	0	0	0	0	0	1	0	0	0
CO3:	Apply numerical techniques to perform various mathematical task such as solving equations, interpolation, integration and curve fitting	3	0	0	0	0	0	0	0	1	0	0	0
CO4:	Solve Brachistochrone, shortest distance, minimal surface area and hanging chain problems and to find the geodesics of known surfaces using Euler-Lagrange method.	3	0	0	0	0	0	0	0	0	0	0	0
CO5:	Evaluate line and surface integrals using Green's, Stoke's and Gauss divergence theorems which have its application in computing the amount of work done, area and volume.	3	0	0	0	0	0	0	0	0	0	0	0
CO6:	Estimate the strength of the relationship between the variables using correlation coefficients and to express the relationship in the form if an equation using regression.	3	0	0	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

CV A,B
IS, B

Solution IAT-3

A necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y')\,dx$ where $y(x_1) = y_1$ & $y(x_2) = y_2$ to be an extremum is that

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \quad \text{(Euler's eqn)}$$

Proof:

Let I be an extremum along some curve $y = y(x)$ passing through $P(x_1, y_1)$ & $Q(x_2, y_2)$.

Also let $y = y(x) + h\alpha(x)$ be the neighbouring curve (where h is small) joining these points. $\therefore \alpha(x_1) = 0$ at P & $\alpha(x_2) = 0$ at Q.

When $h = 0$ these 2 curves coincide thus making I extremum.

$$I = \int_{x_1}^{x_2} f(x, y(x) + h\alpha(x), y'(x) + h\alpha'(x))\,dx.$$

is an extremum when $h = 0$

$\Rightarrow \frac{dI}{dh} = 0$ when $h = 0$, I function of h

by Leibnitz rule

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \frac{\partial}{\partial h} f(x, y(x) + h\alpha(x), y'(x) + h\alpha'(x))\,dx.$$

Apply chain rule $\frac{dI}{dh} = \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial x}\frac{\partial x}{\partial h} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial h} + \frac{\partial f}{\partial y'}\frac{\partial y'}{\partial h}\right) dx$

$\frac{\partial x}{\partial h} = 0$ ($h \to$ independent of x)

$\therefore y' = y'(x) + h\,\alpha'(x)$

& $\frac{\partial y}{\partial h} = \alpha(x)$ & $\frac{\partial y'}{\partial h} = \alpha'(x)$

$$\therefore \frac{dI}{dh} = \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y}\alpha(x) + \frac{\partial f}{\partial y'}\alpha'(x) \right) dx$$

Integrati IInd term by using Integration by parts

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \frac{\partial f}{\partial y}\alpha(x)\,dx + \left(\frac{\partial f}{\partial y'}\alpha(x_2) - \frac{\partial f}{\partial y'}\alpha(x_1) \right) - \int_{x_1}^{x_2} \alpha(x)\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) dx$$

$\alpha(x_1) = 0$ & $\alpha(x_2) = 0$

$$\therefore \frac{dI}{dh} = \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) \right) \alpha(x)\,dx$$

$\frac{dI}{dh}$ must be zero when $h = 0$ for I to be an extremum

$\therefore$ The integrand must be zero

since $\alpha(x)$ is arbitrary

$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0.$ is the required

Euler equation.

2) $f(30) = -30 \quad f(34) = -13, \quad f(38) = 3, \quad f(42) = 18$

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)x_0}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} + \frac{(y-y_0)(y-y_2)(y-y_3)x_1}{(y_1-y_0)(y_1-y_2)(y_1-y_3)}$$

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)x_2}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} + \frac{(y-y_0)(y-y_1)(y-y_2)x_3}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}$$

$$x(0) = \frac{(13)(-3)(-18)30}{(-17)(-33)(-48)} + \frac{30(-3)(-18)(34)}{(17)(-16)(-31)} + \frac{30(13)(-18)(38)}{(33)(16)(-15)}$$

$$+ \frac{(30)(13)(-3)42}{(48)(31)(15)}$$

$$= -0.7821 + 6.5322 + 33.6818 - 2.2016$$

$$x(0) = 37.2303$$

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3) Find $y(1.4)$

x	1	2	3	4	5
y	10	26	58	112	194

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	10				
		16			
2	26		16		
		32		6	
3	58		22		0
		54		6	
4	112		28		
		82			
5	194				

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \dots$$

$$r = \frac{x - x_0}{h} = \frac{1.4 - 1}{1} = 0.4$$

$\Delta y_0 = 16$, $\quad \Delta^2 y_0 = 16$, $\quad \Delta^3 y_0 = 6$, $\quad \Delta^4 y_0 = 0$

$$y(1.4) = f(1.4) = 10 + (0.4)16 + \frac{(0.4)(0.4-1)}{2}(16) + \frac{(0.4)(0.4-1)(0.4-2)}{6}(6)$$

$$y(1.4) = 14.864$$

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4) Correlation coefficient.

x	y	$z = x - y$	x^2	y^2	z^2
1	2	-1	1	4	1
2	5	-3	4	25	9
3	3	0	9	9	0
4	8	-4	16	64	16
5	7	-2	25	49	4
$\Sigma x = 15$	$\Sigma y = 25$	$\Sigma z = -10$	$\Sigma x^2 = 55$	$\Sigma y^2 = 151$	$30 = \Sigma z^2$

$$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = \frac{55}{5} - (3)^2 = 2$$

$$\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2 = \frac{151}{5} - (5)^2 = 5.2.$$

$$\sigma_z^2 = \frac{\Sigma z^2}{n} - (\bar{z})^2 = \frac{30}{5} - (-2)^2 = 2.$$

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_z^2}{2\sigma_x \sigma_y} = \frac{2 + 5.2 - 2}{2\sqrt{2}\sqrt{5.2}} \sim 0.81$$

$$(y - \bar{y}) = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$y - 5 = (0.81) \frac{\sqrt{5.2}}{\sqrt{2}} (x - 3)$$

$$y = 1.306x + 1.082$$

$$(x - \bar{x}) = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$x - 3 = (0.81) \frac{\sqrt{2}}{\sqrt{5.2}} (y - 5)$$

$$x = 0.502y + 0.49$$

5) Find the equation of best fitting st line.

x	y	xy	x^2
5	16	80	25
10	19	190	100
15	23	345	225
20	26	520	400
25	30	750	625
$\Sigma x = 75$	$\Sigma y = 114$	$\Sigma xy = 1885$	$\Sigma x^2 = 1375$

$75a + 5b = 114$

$1375a + 75b = 1885$

$a = 0.7, \quad b = 12.3$

$y = 0.7x + 12.3$ is the equation

~~when~~

6) $x^3 - 4x + 9$, $f(-2) = 9 > 0$, $f(-3) = -6 < 0$

$\therefore$ Negative root lies between $(-3, -2)$

neighbour hood of -3 $f(-2.8) = -1.752$, $f(-2.7) = 0.117$

$\therefore$ I iteration

$a = -2.8$

$b = -2.7$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$= -2.7063$

II iteration

$f(-2.7063) = 0.0041 > 0$

$a = -2.8, \quad b = -2.7063$

$x_2 = -2.7065$

7) $n = 6$ $\quad \int_0^1 \frac{x\,dx}{1+x^2} \rightarrow$ Weddle's rule

$\therefore \quad x = 0,\ 1/6,\ 1/3,\ 1/2,\ 2/3,\ 5/6,\ 1$

$\therefore$

$x = 0 \qquad y_0 = 0$

$x = 1/6 \qquad y_1 = 6/37$

$x = 1/3 \qquad y_2 = 3/10$

$x = 1/2 \qquad y_3 = 2/5$

$x = 2/3 \qquad y_4 = 6/13$

$x = 5/6 \qquad y_5 = \frac{30}{61}$

$x = 1 \qquad y_6 = 1/2$

$$\therefore \int_a^b y\,dx = \frac{3h}{10}\left(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right)$$

$$\int_0^1 \frac{x\,dx}{1+x^2} = 0.3466$$

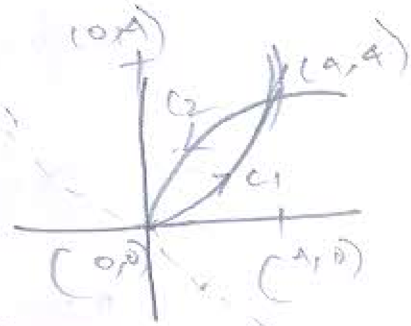
$$\int_0^1 \frac{x}{1+x^2}\,dx = \frac{1}{2}\left(\log_e (1+x^2)\right)_0^1$$

$$= \frac{1}{2}\log_e 2 - \frac{1}{2}\log_e 1$$

$$\int_0^1 \frac{x\,dx}{1+x^2} = \frac{1}{2}\log_e 2$$

$$\Rightarrow \boxed{\log_e 2 = 0.6932}$$

8)



$$\text{Area} = \iint dx\,dy = \frac{1}{2}\int x\,dy - y\,dx$$

$$y^2 = 4x \quad ; \quad x^2 = 4y$$

$$\left(\frac{x^2}{4}\right)^2 = 4x \Rightarrow x\,(x^3 - 64) = 0$$

$$\Rightarrow x = 0, 4$$

$$y = 0, 4$$

point of intersection are $(0,0)$ & $(4,4)$

C_1 is curve $x^2 = 4y \Rightarrow dy = \frac{x}{2}dx$, $0 \le x \le 4$

C_2 is curve $y^2 = 4x \Rightarrow dx = \frac{y}{2}dy$ $4 \le y \le 0$

$$A = \frac{1}{2}\int_{C_1} x\,dy - y\,dx + \frac{1}{2}\int_{C_2} x\,dy - y\,dx$$

$$= \frac{1}{2}\int_{x=0}^{4}\left(x \cdot \frac{x}{2} - \frac{x^2}{4}\right)dx + \frac{1}{2}\int_{y=4}^{0}\left(\frac{y^2}{4} - \frac{y^2}{2}\right)dy$$

$$= \frac{1}{2}\int_{x=0}^{4}\frac{x^2}{4}\,dx - \frac{1}{2}\int_{y=0}^{4} -\frac{y^2}{4}\,dy$$

$$= \left(\frac{x^3}{24}\right)_0^4 + \left(\frac{y^3}{24}\right)_0^4$$

Ans. $= \frac{64}{24} + \frac{64}{24} = \frac{16}{3}$ sq. units