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Internal Assessment Test 1 – September 2016

Sub:	Discrete Mathematical Structures					Code:	15CS36
Date:	8/09/16	Duration:	90 mins	Max Marks:	50	Sem: 3	Branch: ISE- A&B CS-A,B&C

Q1. This question is compulsory. [1x8]

Define a Tautology and a contradiction. Examine whether the compound proposition $[(p \vee q) \rightarrow r] \leftrightarrow [\neg r \rightarrow \neg(p \vee q)]$ is a tautology.

Answer any six of Q2 to Q8. [6x7]

Q2. Verify the principle of duality for the following logical equivalence:

$$[\neg(p \wedge q) \rightarrow \neg p \vee (\neg p \vee q)] \Leftrightarrow (\neg p \vee q).$$

Q3. Write the following propositions in symbolic form and find their negation:

(i) For all x if x is odd then $x^2 - 1$ is even.

(ii) All the integers are rational numbers and some rational numbers are not integers.

Q4. Give a direct proof, an indirect proof and proof by contradiction for the following statement.

"If m is an even integer then m+7 is an odd integer."

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Q5. Test the validity of the following argument:

$$\begin{array}{l} p \to q \\ q \to (r \wedge s) \\ \neg r \vee (\neg t \vee u) \\ \quad p \wedge t \\ \hline \therefore u \end{array}$$

Q6. Let p(x): $x^2 - 8x+15 = 0$, q(x): x is odd, r(x): x>0 with the set of all integers as the universe. Find the truth values of the following statements. If a statement is false, give a counter example

(i) $\forall x, [\{p(x) \vee q(x)\} \to r(x)]$, (ii) $\forall x, [q(x) \to p(x)]$, (iii) $\exists x, [p(x) \to \{q(x) \wedge r(x)]$,

(iv) $\forall x, [\neg q(x) \to \neg r(x)]$

Q7. Define the Well-ordering principle. Using Mathematical induction, prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$.

Q8. Find an explicit formula for $a_1 = 7, a_n = 2a_{n-1} + 1$ for $n \geq 2$.

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Q1. A compound proposition is said to be a tautology if it is true for all combinations of truth values of its components.

A compound proposition is said to be a contradiction if it is false for all combinations of truth values of its components.

p	q	r	$p\vee q$	$(p\vee q)\to r$	$\neg r$	$\neg(p\vee q)$	$\neg r\to\neg(p\vee q)$	$[(p\vee q)\to r]\leftrightarrow[\neg r\to\neg(p\vee q)]$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	0	1	1	1
0	1	0	1	0	1	0	0	1
0	1	1	1	1	0	0	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	0	1	0	0	1
1	1	1	1	1	0	0	0	1

Since all the entries of last column are 1,

$[(p\vee q)\to r]\leftrightarrow[\neg r\to\neg(p\vee q)]$ is a tautology.

Q2. Let $u = [\neg(p \wedge q) \rightarrow \neg p \vee (\neg p \vee q)]$

$$\Leftrightarrow (p \wedge q) \vee [\neg p \vee (\neg p \vee q)]$$ $p \rightarrow q \Leftrightarrow \neg p \vee q$

$u^d = (p \vee q) \wedge [\neg p \wedge (\neg p \wedge q)]$

$\Leftrightarrow (p \vee q) \wedge [(\neg p \wedge \neg p) \wedge q]$ associative law

$\Leftrightarrow (p \vee q) \wedge (\neg p \wedge q)$ Idempotent law

$\Leftrightarrow [(p \vee q) \wedge q] \wedge \neg p$ commutative and associative law

$\Leftrightarrow q \wedge \neg p$ absorption law

$\Leftrightarrow \neg p \wedge q$ —— ① commutative law

Let $v = \neg p \vee q$

$\therefore v^d = \neg p \wedge q$ —— ②

From ① & ②, $u^d \Leftrightarrow v^d$.

Hence Principle of duality is verified.

Q3.

(i) Let $p(x)$: x is odd.

$q(x)$: $x^2 - 1$ is even.

Given $\forall x, \; p(x) \rightarrow q(x)$

Neg^n: $\neg\left[\forall x,\ p(x) \rightarrow q(x)\right]$

$\Leftrightarrow \exists x,\ \neg\left[p(x) \rightarrow q(x)\right]$

$\Leftrightarrow \exists x,\ p(x) \wedge \neg q(x)$

ie Some numbers are odd and not even.

(ii) Q: set of all rational no

Z: set of all integers

$p(x)$: x is a rational no

$q(x)$: x is an integer.

$\{\forall x \in Z,\ p(x)\} \wedge \{\exists x \in Q,\ \neg q(x)\}$

Neg^n: $\neg\{\forall x \in Z,\ p(x)\} \vee \neg\{\exists x \in Q,\ \neg q(x)\}$

$\Leftrightarrow \{\exists x \in Z,\ \neg p(x)\} \vee \{\forall x \in Q,\ q(x)\}$

ie Some integers are not rational numbers ~~~~ or all rational numbers are integers.

Let p: m is an even no

q: $m+7$ is an odd no

Given $p \rightarrow q$

Direct Proof:

Let m be an even no.

ie $m = 2k \quad \forall k \in Z$

Then, $m+7 = 2k+7$

$= 2k+6+1$

$= 2(k+3)+1$

$= 2n+1 \qquad k+3 = n$

which is an odd no

Hence the proof.

Indirect Proof:- $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$

Let $\neg q$ be true.

$\Rightarrow m+7$ is not odd.

$\Rightarrow m+7$ is even.

Let $m+7 = 2p$

$m = 2p-7 = 2(p-3)-1 = 2q-1$

which is an odd no $\qquad q = p-3$

$\Rightarrow \neg p$ is true.

Let us assume that the given statement $p \to q$ is false.

ie p is true and q is false

m is an even no and $m+7$ is not an odd no

$\Rightarrow m+7$ is even.

$\Rightarrow m+7 = 2k \quad k \in Z$

$\Rightarrow m = 2k-7$

$\Rightarrow m = 2(k-3)-1$

$= 2p-1 \quad p = k-3 \in Z$

which is an odd.

This is a contradiction.

This is because of our wrong assumption.

$\therefore$ Given statement is true.

Q5.

$$\begin{array}{l} p \to q \\ q \to (r \wedge s) \\ \neg r \vee (\neg t \vee u) \\ \underline{p \wedge t} \\ \therefore u \end{array} \quad \Rightarrow \quad \begin{array}{l} p \to q \\ q \to (r \wedge s) \\ \neg r \vee (\neg t \vee u) \\ p \\ \underline{t} \\ \therefore u \end{array}$$

Rule of conjunctive simplification

q Using Modus Pones for I & IV premises.

$q \to (r \wedge s)$

$\Rightarrow \neg r \vee (\neg t \vee u)$

t

$\therefore u$

$\Rightarrow$ $r \wedge s$ Modus Pones for I & II.

$\neg r \vee (\neg t \vee u)$

t

$\therefore u$

$\Rightarrow$ s Rule of conjunctive simplification

r

$\neg r \vee (\neg t \vee u)$

t

$\therefore u$

$\Rightarrow$ $\neg t \vee u$ Using disjunctive syllogism for 2nd & 3rd

t

$\therefore u$

This is valid argument in view of disjunctive syllogism.

Q6. $p(x): x^2 - 8x + 15 = 0$; $x = 3, 5$

$q(x)$: x is odd

$r(x): x > 0$

(i) $\forall x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$

FALSE

For $x = -1$, $q(x)$ is true.

$\therefore p(x) \vee q(x)$ is true.

But $r(x)$ is false.

$\because x = -1 < 0$

(ii) $\forall x, [q(x) \rightarrow p(x)]$

FALSE

For $x = 7$, $q(x)$ is true.

But $p(x)$ is false.

(iii) $\exists x, [p(x) \rightarrow \{q(x) \wedge r(x)\}]$

TRUE

(iv) $\forall x, [\neg q(x) \rightarrow \neg r(x)]$

FALSE

Eg:- $x = 2$ $\neg q(x)$ is true

$\neg r(x)$ is false.

Q7. Well-Ordering Principle – "Every non-empty subset of Z^+ contains a smallest element."

Let $S(n): 4n < (n^2-7)$

Basis step: For $n=6$

$4 \cdot 6 < 6^2 - 7$ which is true.

$\therefore S(6)$ is true.

Induction step:

Assume that $S(k)$ is true for $k \geq 6$.

$4k < (k^2-7)$ —— (1)

Consider $4(k+1) = 4k+4$

$< (k^2-7)+4$ wkt $4 < 2k+1$ $\forall k \geq 6$

$< (k^2-7)+(2k+1)$

$= (k+1)^2 - 7$

$\therefore 4(k+1) < (k+1)^2 - 7$

The result is true for $n = k+1$.

$\therefore$ By Mathematical Induction, $S(n)$ is true for all positive integers n.

Q8.

$$a_n = 2a_{n-1} + 1$$

$$= 2(2a_{n-2} + 1) + 1$$

$$= 2^2 a_{n-2} + 2 + 1$$

$$= 2^2 (2a_{n-3} + 1) + 2 + 1$$

$$= 2^3 a_{n-3} + 2^2 + 2 + 1$$

$$= 2^3 (2a_{n-4} + 1) + 2^2 + 2 + 1$$

$$= 2^4 a_{n-4} + 2^3 + 2^2 + 2 + 1$$

$$\vdots$$

$$= 2^{n-1} a_{n-(n-1)} + 2^{n-2} + 2^{n-3} + \cdots + 2^2 + 2 + 1$$

$$= 2^{n-1} a_1 + (1 + 2 + 2^2 + \cdots + 2^{n-2})$$

$$\left[1 + r + r^2 + \cdots + r^{n-1} = \frac{r^n - 1}{r - 1} \right.$$
$$\left. \text{sum of } n \text{ terms of G.S} \right]$$

$$= 7 \times 2^{n-1} + \frac{2^{n-1} - 1}{2 - 1} = 8 \times 2^{n-1} - 1$$

$$= 2^{n+2} - 1$$