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**Internal Assessment Test I– September 2016**

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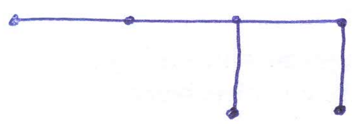
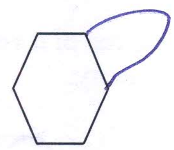
| **Reg. #** | | | | |
| Sub | Discrete Mathematical Structures | | **Code** | 15CS36 |
| Date: 08/09/ 2016 | Duration: 90 mins | Max Marks: 50 | Sem 3 | **Branch** CSE D |

**Note: First question (8 marks) is compulsory. Answer six questions from the rest. (6×7=42)**

1. Define isomorphism between two graphs. Find whether the following two graphs are isomorphic or not.



2. State Hand shaking property. Prove that in every graph the number of vertices of odd degree is even.
3. Discuss Konigsberg bridge problem with a decent sketch of diagrams.
4. Define Euler's Trail and Euler's circuit. Give an example each for the graph.



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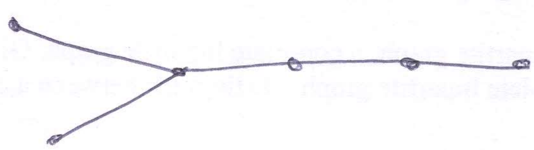

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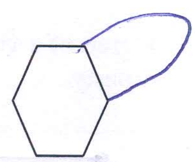
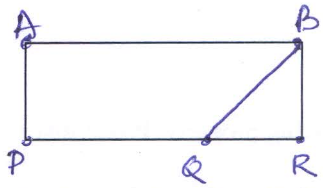


5.Prove that a k-dimensional hypercube $Q_k$ has $k(2^{k-1})$ edges. Find the number of edges in $Q_8$. What is the dimension of the hypercube with 524288 edges?

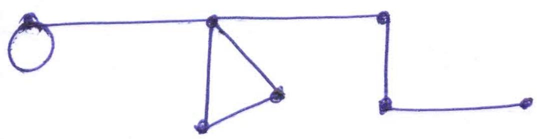
6.Define a subgraph, a spanning subgraph, and an induced subgraph.

7.Define a bipartite graph, a complete bipartite graph. Give an example of a graph which is bipartite but not complete bipartite graph. Distinguish between a complete graph and a complete bipartite graph.

8. Define a path. Find all the paths from A to R. Find their lengths.




9.Define degree of a vertex in a graph. Find the degree of each vertex in the graph and verify hand shaking property.



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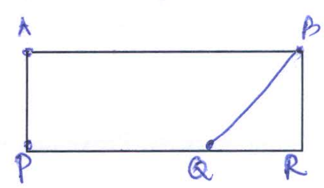


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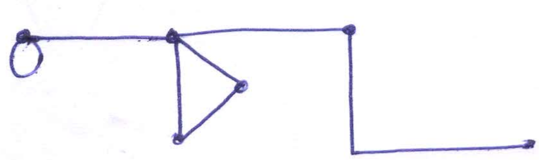
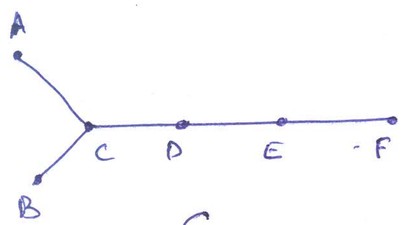


3D - CSE - DMS - IAT-I Scheme & Solutions

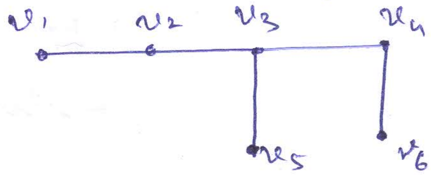
1. Let $G = (V, E)$ & $G'(V', E')$ be two given graphs. If $f : V \to V'$ is a bijection $\ni$ i) $f$ is a one-one correspondence

ii) $\forall$ vertices $A, B \in G$, $\{A, B\}$ is an edge of $G$ iff $\{f(A), f(B)\}$ is an edge of $G'$.

(or) $f$ is an isomorphism if $\exists$ a 1-1 correspondence b/w their vertices, b/w their edges $\ni$ adjacency of vertices is preserved. → (2)




G




G'

| | G | G' | |
|---|---|---|---|
| 1) | no of vertices = 6 | no of vertices = 6 | → (1) |
| 2) | " edges = 5 | no of edges = 5 | → (1) |
| 3) | no of vertices of deg 1 = 3 | no of vertices of deg 1 = 3 | (2) |
| | " deg 2 = 2 | " " deg 2 = 2 | |
| | " deg 3 = 1 | " " deg 3 = 1 | |
| 4) | Vertices adj to C (of deg 3) are of deg 1, 1, 2; | Vertices adjacent to $v_3$ (of deg 3) are of deg 2, 1, 2. | (2) |

∴ G & G' are not isomorphic as adjacency is not preserved.

2. Hand shaking property :- The sum of the degrees of all vertices in a graph is an even number = 2 (no of edges in the graph). → (2)

Thm :- In every graph, the no of vertices of odd degree is even.

Pf: Let G be a graph with $n$ vertices.
Let $k$ be the no of vertices of odd degree

$\Rightarrow (n-k)$ vertices are of even degree

Let $v_1, v_2, \ldots v_k$ be the vertices of odd degree

$v_{k+1}, v_{k+2} \ldots v_n$ " " even " $\longrightarrow$ (2)

Then sum of ~~ver~~ degrees of all vertices is given by

$$\sum_{i=1}^{n} \deg(v_i) = \sum_{i=1}^{k} \deg(v_i) + \sum_{i=k+1}^{n} \deg(v_i).$$

$$\Rightarrow 2|E| = \sum_{i=1}^{k} \deg(v_i) + \sum_{i=k+1}^{n} \deg(v_i)$$

even no = " + sum of even numbers

$$\Rightarrow \sum_{i=1}^{k} \deg(v_i) = \text{An even no} - \text{even no.} = \text{even}$$

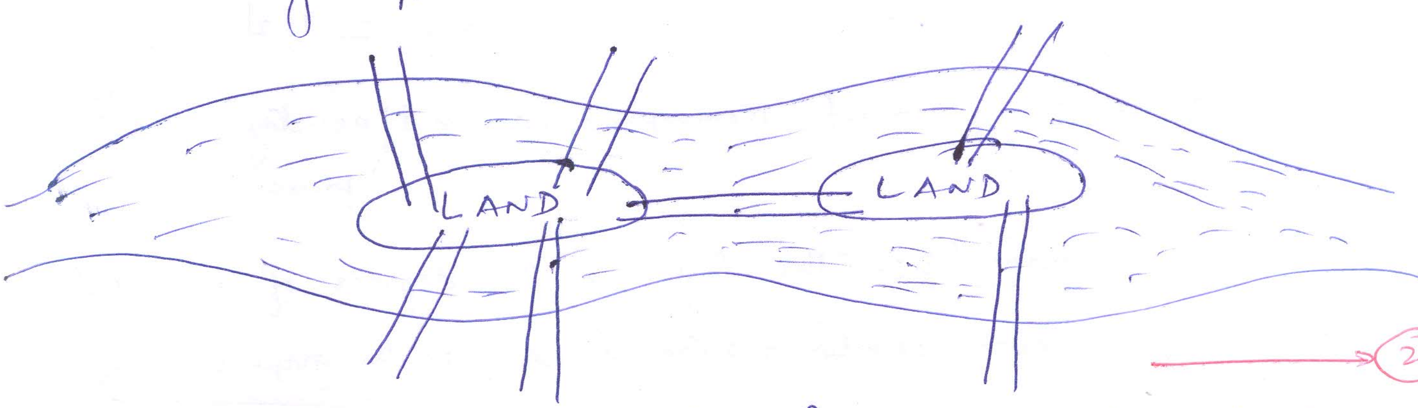
$\because$ $\deg(v_i)$ is odd $\forall i = 1$ to $k$, $k$ has to be even for $\longrightarrow$ (3)

the sum to be even.

Hence the proof.

(3) Discuss konigs berg bridge problem.

Problem:- • Starting from any landpoint travel thro' all the bridges (but only once) & come back to the starting point.

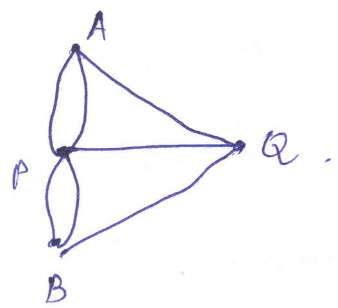



$\longrightarrow$ (2)

Konigsberg is a city in East Prussia, thro' which the river Pigel flowed dividing the city into 4 land parts

In 1736, Euler analysed this problem with the help of a graph & gave the solution. The Soln is as given below

Land areas are considered as vertices A, B, P & Q where A, B are banks of the river & P, Q are islands. A graph is constructed with these vertices & 7 bridges as edges. It can be seen as




Here deg (A) = deg (B) = deg Q = 3

deg (P) = 5 → 3

All the degrees are odd.

∵ A connected graph is an Euler's graph iff all vertices are of even degree.

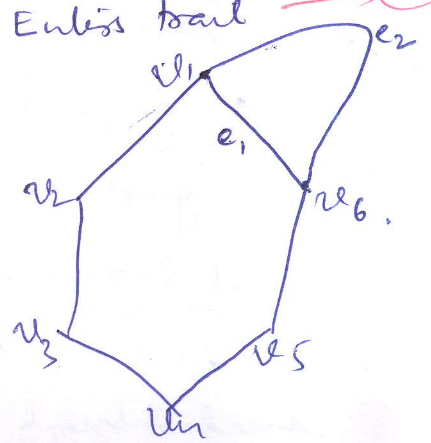
⇒ This graph has no Euler's circuit

⇒ It is not possible to start at any land area & cross all 7 bridges (only once) but come back to starting point. → 2

4.

Euler's Circuit:- If there is a circuit in a connected graph G that contains all the edges of G, then it is an Euler's circuit. → 2

If there is a trail in a connected graph G that contains all the edges of G, then it is an Euler's trail → 2




Euler's trail :- $v_1 e_1 v_6 e_2 v_1 v_2 v_3 v_4 v_5 v_6$ → 2

Euler's circuit :- has no Euler circuit if

$\deg(v_1) = 3 = \deg(v_6)$ → 1

(5) We know no of vertices in a hypercube $Q_k$ is $2^k$.

& degree of each vertex is $k$.

$\therefore$ Sum of degrees of vertices of $Q_k$ is $k \times 2^k$.

from hand shaking property, $k \times 2^k = 2|E| \Rightarrow |E| = \frac{k \times 2^k}{2} = k2^{k-1}$

$\therefore$ No of edges in a hypercube $Q_k$ is $k2^{k-1}$. (3)

No of edges in $Q_8 = 8\,2^{8-1} = 8 \times 2^7 = 8 \times 128 = 1024$. (2)

Dimension of hypercube $Q_k$ with 524288 edges.

$\Rightarrow k2^{k-1} = 524288$

$\Rightarrow 16 \times 2^{15} = 524288 \Rightarrow k = 16$. (2)

So dimension = 16

(6) Given two graphs $G_1$ & $G_2$, we say $G_1$ is a subgraph of $G_1$

if (i) all vertices of $G_2$ are vertices of $G_1$

(ii) " edges " edges of $G_1$

iii) each edge of $G_2$ has the same end vertices in $G_1$ as in $G_2$.

$\therefore$ a sub graph is a part of another graph (3)

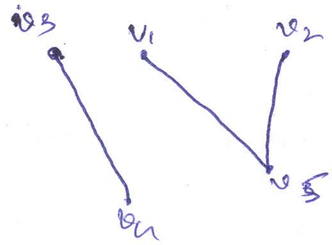
Spanning Subgraph :- Given $G = (V, E)$, if $\exists$ a subgraph $G_1 = (V_1, E_1)$ of $G$ such that $V_1 = V$ then $G_1$ is a spanning subgraph of $G$.

A graph & all its spanning subgraphs have same vertex set

Each graph is a spanning subgraph of itself (2)

Induced Subgraph :- Given $G = (V, E)$, if $\exists$ a subgraph $G_2 = (V_2, E_2)$ of $G$ such that each of of $G$ is an edge of $G_1$ also. then $G_1$ is an induced subgraph of $G$. (2)

(7) **Bipartite graph** :- Let $G$ be a simple graph $\ni$ its vertex set $V$ is the union of two mutually disjoint non-empty subsets $V_1$ & $V_2$ $\ni$ each edge in $G$ joins a vertex $V_1$ & a vertex in $V_2$. Then $G$ is called a bipartite graph. If $E$ is the edge set, it is denoted by $G = (V_1, V_2, E)$. Here $V_1$ & $V_2$ are bipartites of the vertex set $V$. eg

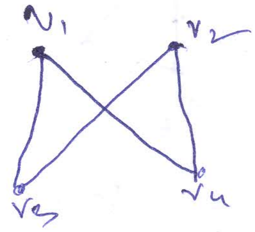



$V_1 = \{v_1, v_2, v_3\}$.

$V_2 = \{v_4, v_5\}$

2

**Complete bipartite graph** :- A bipartite graph $G = (V_1, V_2, E)$ is called a complete bipartite graph if $\exists$ an edge b/w every vertex of $V_1$ & every vertex in $V_2$.

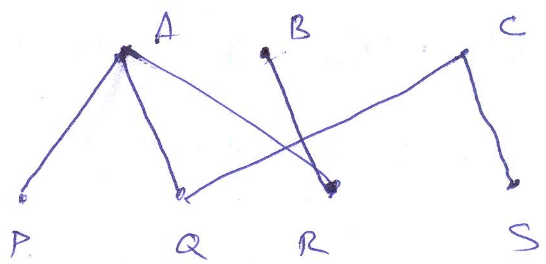



$V_1 = \{v_1, v_2\}$

$V_2 = \{v_3, v_4\}$.

2

Example of a bipartite graph which is not complete bipartite graph is




1

A complete bipartite graph is not a complete graph b'cos in a complete graph $\exists$ an edge b/w every pair of vertices of $G$. Where as in a complete bipartite graph $\exists$ an b/w the every vertex ~~set~~ element of $V_1$ & any element of $V_2$ but not b/w elements of $V_1$ or elements of $V_2$.

2

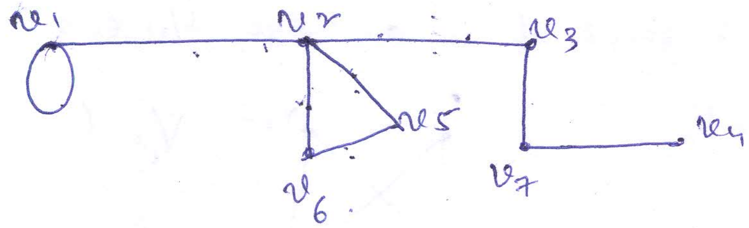
(8) Path :- It is a trail (a ~~the~~ walk in which no edge appears more than once) in which no ~~ap~~ vertex appears more than once. (2)

Paths from A to R

i) ABR  ii) ABQR  iii) APQR.  iv) APQBR. (3)

length 2  length 3  length 3  length 4. (2)

(9) The no of edges incident ~~of~~ a graph G that are incident on a vertex $v$ of G with loops counted twice is called the degree of $v$ in G. It is denoted by deg($v$). (2)




deg($v_1$) = 3 (loop1 + edge1) | deg($v_3$) = 2 | deg($v_5$) = 2

deg($v_2$) = 4 | deg($v_4$) = 1 | deg($v_6$) = 2

deg($v_7$) = 2 (2)

Hand shaking property : The sum of degrees of all vertices in a graph is an even number = 2 (no of edges)

∴ Sum of degrees of all vertices of G = 3 + 4 + 2 + 1 + 2 + 2 + 2 = 16

No of edges = 8

∴ Hand shaking property is verified. (3)