CMR INSTITUTE OF TECHNOLOGY

USN

Internal Assesment Test - II

Sub:	DISCRETE MATHEMATICAL STRUCTURES							Code:	15CS36
Date:	04 / 11 / 2016	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	CS, IS

Question 1 is compulsory and answer any six from questions 2 to 8.

		Marks	OBE CO	OBE RBT
1	Draw the Hasse diagram for the relation R defined by aRb iff a divides b, on the set of positive divisors of 45. Write the relation R.	[08]	CO3	L1
2 (a)	Find the coefficient of $w^3x^2yz^2$ in the expansion of $(2w-x+3y-2z)^8$. 32	[03]	CO2	L3
(b)	Determine the number of integer solution of the equation $x_1+x_2+x_3+x_4=15$, where x_1, $x_2 \geq 5$, x_3, $x_4 \geq 7$.	[04]	CO2	L3
3	Let $A=\{1,2,3,4,5\}$. Define a relation R on $A\times A$ by $(x_1,y_1)R(x_2,y_2)$ iff $x_1+y_1=x_2+y_2$. (i) Verify that R is an equivalence relation on $A\times A$. (ii) Determine the partition of $A\times A$ induced by R.	[07]	CO3	L3
4 (a)	Let $f: R \to R$ be defined by $f(x)=\begin{cases} 3x-5, & for\ x>0 \\ -3x+1, & for\ x\leq 0 \end{cases}$	[07]		
(i)	Determine $f(-1), f(5/3)$.		CO3	L3
(ii)	Find $f^{-1}(-1), f^{-1}(-6)\ and\ f^{-1}([-5,5])$.		CO3	L3
	OR			
(b)	Let $A=\{x \mid x\ is\ real\ and\ x\geq -1\}$, and $B=\{x \mid x\ is\ real\ and\ x\geq 0\}$. Consider the function $f: A\to B$ defined by $f(a)=\sqrt{a+1}$, for all $a\in A$. Show that f is invertible and determine f^{-1}.			
5	If 10 points are selected from the interior of a triangle whose sides are of length 3 cms (each), show that at least two points are within 1 cm apart.	[07]	CO3	L1
6	Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party find that only one chair at each of five tables T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4, and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs.	[07]	CO6	L3
7	The number of virus affected files in a system is 1000 (to start with) and this increases by 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day.	[07]	CO6	L3
8	Solve the recurrence relation $a_{n+2}^2-5a_{n+1}^2+4a_n^2=0\ for\ n\geq 0$, Given $a_0=4\ and\ a_1=13$.	[07]	CO6	L3

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Verify the correctness of an argument using propositional and predicate logic and truth tables.	3	2	2	1	1	1	0	1	1	1	0	1
CO2:	Solve problems using counting techniques and combinatorics in the context of discrete probability.	3	2	2	1	1	1	0	1	1	1	0	1
CO3:	Solve problems involving relations and functions and their properties.	3	2	2	1	1	1	0	1	1	1	0	1
CO4:	Construct proofs using direct proof, proof by contradiction, proof by contradiction, and proof by cases and Mathematical induction.	3	2	2	1	1	1	0	1	1	1	0	1
CO5:	Explain and differentiate graphs and trees.	3	2	2	1	1	1	0	1	1	1	0	1
CO6:	Solve problems involving recurrence relations and generating functions.	3	2	2	1	1	1	0	1	1	1	0	1

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

1

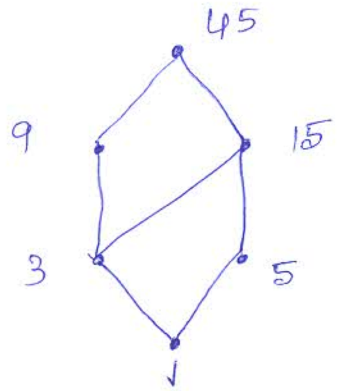
Discrete Mathematical Structures - 15CS36

IAT - II

1. Let A be the set of all positive divisors of 45.

$A = \{1, 3, 5, 9, 15, 45\}$ — (1)

$R = \{(1,3), (1,5), (1,9), (1,1), (1,15), (1,45), (3,3)$
$(3,9), (3,15), (3,45), (5,5), (5,15), (5,45)$
$(9,9), (9,45), (15,15), (15,45), (45,45)\}$
— (2)



— (5)

2(a) The general term ~~of~~ in the expansion of $(2w-x+3y-2z)^8$

is $\binom{8}{n_1\ n_2\ n_3\ n_4}(2w)^{n_1}(-x)^{n_2}(3y)^{n_3}(-2z)^{n_4}$ — (1)

For $n_1 = 3$, $n_2 = 2$, $n_3 = 1$, $n_4 = 2$, this becomes

$$\binom{8}{3\ 2\ 1\ 2}(2w)^3(-x)^2(3y)(-2z)^2$$ — (1)

$$= \frac{8!}{3!\,2!\,1!\,2!}\, 2^3 \times (-1)^2 \times 3 \times (-2)^2\, w^3x^2yz^2$$ — (1)

$\therefore$ Req coeff^t is 161280

2 (b) $x_1+x_2+x_3+x_4=32$ — ①

Let $x_1, x_2 \geq 5$, $x_3, x_4 \geq 7$

$y_1 = x_1-5$, $y_2 = x_2-5$, $y_3 = x_3-7$, $y_4 = x_4-7$ — (1)

Now $y_1, y_2, y_3, y_4 \geq 0$

① becomes

$(y_1+5)+(y_2+5)+(y_3+7)+(y_4+7) = 32$ — (1)

$\Rightarrow y_1+y_2+y_3+y_4 = 32-14-10 = 32-24 = +8$

$n=4$, $r=8$

Required no is $C(n+r-1, r) = C(4+8-1, 8)$ — (1)

$= C(11, 8)$ — (1)

3 (i) For all $(x,y) \in A \times A$, we have $x+y = x+y$!

ie $(x,y) R (x,y)$.

$\therefore$ R is reflexive. — (1)

Next take any $(x_1,y_1), (x_2,y_2) \in A \times A$

& let $(x_1,y_1) R (x_2,y_2) \Rightarrow x_1+y_1 = x_2+y_2 \Rightarrow x_2+y_2 = x_1+y_1$

$\Rightarrow (x_2,y_2) R (x_1,y_1)$

$\therefore$ R is symmetric. — (1)

Now take any (x_1,y_1) (x_2,y_2), $(x_3,y_3) \in A \times A$

Suppose $(x_1,y_1) R (x_2,y_2)$ and $(x_2,y_2) R (x_3,y_3)$

②

$\Rightarrow x_1+y_1 = x_2+y_2$ & $x_2+y_2 = x_3+y_3$

$\Rightarrow x_1+y_1 = x_3+y_3$

$\Rightarrow (x_1, y_1) R (x_3, y_3)$

$\therefore$ R is transitive. — (1)

& hence R is an equivalence relⁿ

$[(1,1)] = \{(1,1)\} = A_1$ (say)

$[(1,2)] = \{(1,2), (2,1)\} = [(2,1)] = A_2$

$[(1,3)] = \{(1,3), (3,1), (2,2)\} = [(2,2)] = [(3,1)] = A_3$

$[(1,4)] = \{(1,4), (2,3), (3,2), (4,1)] = [(2,3)] = [(3,2)] = [(4,1)] = A_4$

$[(1,5)] = \{(1,5), (2,4), (3,3), (4,2), (5,1)\} = [(2,4)] = [(3,3)] = [(4,2)] = [(5,1)] = A_5$

$[(2,5)] = \{(2,5), (3,4), (4,3), (5,2)\} = [(3,4)] = [(4,3)] = [(5,2)] = A_6$

$[(3,5)] = \{(3,5), (4,4), (5,3)\} = [(4,4)] = [(5,3)] = A_7$

$[(4,5)] = \{(4,5), (5,4)\} = [(5,4)] = A_8$

$[(5,5)] = \{(5,5)\} = A_9$

Thus $P = \{A_1, A_2, A_3 \ldots A_9\}$ is the partition of A because $A_1, A_2 \ldots A_9$ are the only distinct equivalence classes of $A \times A$ wrt R. — (4)

4 a(i) $f(-1) = 3\times(-1)+1 = 4$ —(1)

$f(5/3) = (3\times 5/3) - 5 = 0$ —(1)

$f^{-1}(-1) = \{x\in R \mid f(x) = -1\} = \{4/3\}$

$f(x) \neq -1$ when $x \leq 0$.

—(1)

$f^{-1}(-6) = \phi$ because $f(x) \neq -6$ for any $x \in R$

—(1)

(ii) $f^{-1}([-5,5]) = \{x\in R \mid f(x) \in [-5,5]\}$

$= \{x\in R \mid -5 \leq f(x) \leq 5\}$ —(1)

when $x>0$, we have $f(x) = 3x-5$

$\therefore -5 \leq f(x) \leq 5$

$\Rightarrow -5 \leq (3x-5) \leq 5$

$\Rightarrow 0 \leq x \leq 10/3$

when $x \leq 0$, $-5 \leq f(x) \leq 5$ —(1)

$\Rightarrow -5 \leq -3x+1 \leq 5$

$\Rightarrow -4/3 \leq x \leq 2$

$\therefore f^{-1}([-5,5]) = \{x\in R \mid -4/3 \leq x \leq 2 \text{ or } 0 \leq x \leq 10/3\}$

$= [-4/3, 10/3]$

(1) —(1)

4 (b) Take any $a_1, a_2 \in A$

$\Rightarrow f(a_1) = \sqrt{a_1+1}$, $f(a_2) = \sqrt{a_2+1}$

$f(a_1) = f(a_2)$

$\Rightarrow \sqrt{a_1+1} = \sqrt{a_2+1}$

$\Rightarrow \quad a_1 = a_2$

Hence f is 1-1. —— (3)

Take any $b \in B$. Then $b = f(a)$ holds if $b = \sqrt{a+1}$

$\Rightarrow b^2 = a+1$

$\Rightarrow a = b^2 - 1$.

Since $b \geq 0$, $b^2 - 1 \geq -1$

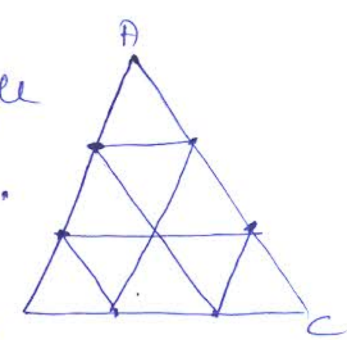
Thus every $b \in B$ has a pre image in A under f.

Hence f is onto as well. ——(4)

$\therefore$ f is invertible.

The inverse of f is given by $f^{-1}(b) = b^2 - 1 \quad \forall\, b \in B$.

5. Let ABC be an equilateral triangle whose sides are of length 3 cm each.



Trisect each side of the Δ ABC —(1) fig

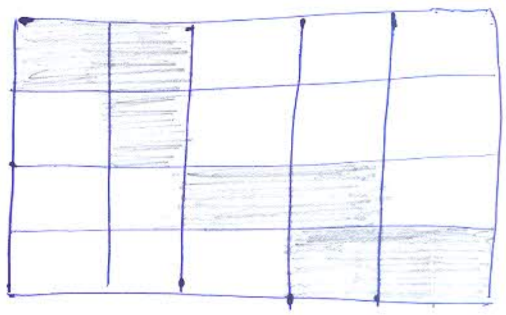
and join the points as shown in the figure.

Now we have 9 small equilateral triangles with their sides equal to 1 cm. Let us treat these triangles as 9 pigeonholes and 10 points taken inside the Δ as pigeons.

By using pigeon hole principle, at least one portion must contain two or more points, the distance b/w such points is less than ½ cm. — (6)

6) Consider the board shown below. Shaded portions represent forbidden places.

For this board made up of shaded squares, the rook polynomial is



— (1)

$$r(C,x) = 1 + 7x + 16x^2 + 13x^3 + 3x^4.$$ — (2)

Here $r_1 = 7$, $r_2 = 16$, $r_3 = 13$, $r_4 = 3$ — (1)

$S_0 = 5! = 120$, $S_1 = 168$, $S_2 = 96$, $S_3 = 26$, $S_4 = 3$ — (2)

Req. no $= S_0 - S_1 + S_2 - S_3 + S_4 = 25$ — (1)

7) Let $a_0 = 1000$ & a_n denote the no of virus affected files after $2n$ hours.

Thus, $a_n = a_{n-1} + a_{n-1} \times \frac{250}{100} = (3.5)\, a_{n-1} \quad n \geq 1$ — (2)

This is the recurrence relation for the no of virus affected files.

Solⁿ: $a_n = (3.5)^n a_0 = 1000 \times (3.5)^n$. — (2)

This gives the no of virus affected files after 2n hours.

From this we get

$a_{12} = 1000 \times (3.5)^{12} = 3379220508$ — (3)

This is the no of virus affected files after 24 hours.

8) Let $b_n = a_n^2$. — (1)

Then the given relⁿ reads

$b_{n+2} - 5b_{n+1} + 4b_n = 0, \; n \geq 0.$

— (1)

$\Rightarrow \quad b_n - 5b_{n-1} + 4b_{n-2} = 0, \; n \geq 2.$

The characteristic eqⁿ is $k^2 - 5k + 4 = 0$

whose roots are $k_1 = 4$ and $k_2 = 1$.

∴ general solⁿ for b_n is $b_n = A \times 4^n + B \times 1^n$ — ①

— (2)

where A and B are arbitrary consts.

Given $a_0 = 4$ & $a_1 = 13$ $\Rightarrow$ $b_0 = 16$ & $b_1 = 169$

— (1) — (1)

We get $16 = A + B$ & $169 = 4A + B$.

Solving $A = 51, \; B = -35$ — (1)

∴ $b_n = 51 \times 4^n - 35$ ∴ $a_n = \pm\sqrt{(51 \times 4^n) - 35}$