### Internal Assessment Test - II

| Sub: | Formal Language and Automata Theory | | | | | Code: | 10CS56 |
|------|-------------------------------------|---|---|---|---|-------|--------|
| Date: | 09 / 11 / 2016 | Duration: | 90 mins | Max Marks: | 50 | Sem: V | Branch: | CSE & ISE |

### Answer Any FIVE FULL Questions

| | | Marks | OBE | |
|---|---|-------|-----|-----|
| | | | CO | RBT |
| 1 | Define a Context free grammar. Design a CFG for the following languages. <br> (i) L={ $ww^r$ \| w is in (a+b)*} <br> (ii) L={ $a^n b^n c^m$ \| n≥0, m>0 } <br> (iii) L={$a^n b^m c^k$ \| k=2m+n, n,m,k≥1} | [1+9] | CO2 | L3 |
| 2 (a) | Define the following terms: <br> (i) Derivation tree (ii) Sentential form (iii) Yield of a tree (iv) Ambiguous grammar (v) Leftmost derivation | [05] | CO2 | L1 |
| (b) | Show that the following grammar is ambiguous for the string s=ibtibtaea. <br> S➡iCtS \| iCtSeS \| a     C➡b | [05] | CO2 | L1 |
| 3 | Consider the following grammar. Generate LMD, RMD and derivation tree(Both LMD and RMD) for the string W=badbabaadb <br> S➡AaAb\|BbBa     A➡aAb\|bAB\|d     B➡aB\|bBa\|∈ | [10] | CO2 | L2 |
| 4 | Design a PDA to accept the language L={$a^n b^{2n}$ \|n≥1} by final state. Also show the moves made by PDA(Instantaneous description) for the string W=aabbbb. | [5+5] | CO3 | L3 |
| 5 (a) | Explain the working model of a PDA with a diagram. | [5] | CO3 | L1 |
| (b) | Obtain the empty stack PDA for the following grammar <br> S➡aAc\| aSc     A➡bAc\|bc | [5] | CO3 | L2 |
| 6 (a) | Define the following terms: <br> (i) Generating Symbol (ii) Reachable Symbol (iii) Unit Production <br> (iv) Null Production | [4] | CO2 | L2 |

| | | | | |
|---|---|---|---|---|
| (b) | Consider the following grammar<br>S➡ABC \| BaB     A➡aA\|BaC\|aaa     B➡bBb\|a\|D     C➡CA\|AC<br>    D➡∈<br>Eliminate ∈-productions and useless symbols. | [3+3] | CO2 | L2 |
| 7 | What is Chomsky Normal Form? Convert the following grammar to CNF.<br>E➡T*E \| T - E\|T     T➡T+F \| F     F➡a\|(E) | [10] | CO2 | L2 |
| 8 | Design PDA for the following languages. Is it DPDA or NPDA?<br>(i) L={$wcw^R$ \| w is in {0+1}*, $w^R$ is the reverse of w}<br>(ii)L={$a^n b^m c^{m+n}$\|n,m≥0} | [5+5] | CO3 | L3 |

| Course Outcomes | | PO 1 | PO 2 | PO 3 | PO 4 | PO 5 | PO 6 | PO 7 | PO 8 | PO 9 | PO 10 | PO 11 | PO 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CO 1: | Apply techniques of induction, Deduction, Contradiction to formally prove simple theorems. | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| CO 2: | Construct and apply grammars for simple languages – Regular, CFG. | 0 | 1 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| CO 3: | Model and solve simple classes of problems – Regular, CFG, RE class of problems. | 0 | 1 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |

| CO 4: | Apply pumping lemma to determine if a language is regular, CFG or neither. | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CO 5: | Model and Construct turing machine as a solution for simple problems. | 0 | 1 | 2 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

| Cognitive level | KEYWORDS |
|---|---|
| L1 | List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc. |
| L2 | summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend |
| L3 | Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover. |
| L4 | Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer. |
| L5 | Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize. |

1) Define —— 1 mark.

   i) $S \longrightarrow aaS_1b \mid aab$ —— 2 marks Grammar, 1 mark correct termination.

   ii) $S \longrightarrow '('S')' \mid '\{'S'\}' \mid '['S']' \mid SS \mid \varepsilon$ —— "

   iii) $S \longrightarrow aSc \mid aS_1c$
      $S_1 \longrightarrow bS_1c \mid bc$ —— "

2) a)   1 × 5   marks
   b)   1 × 4   marks + 1 # if all correct

3)
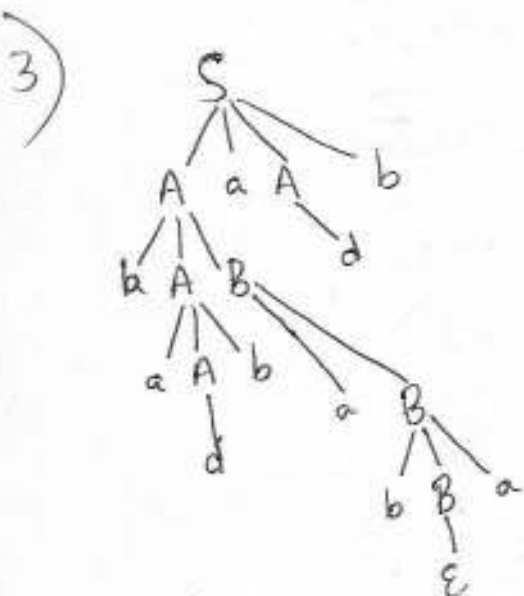

Tree can be same for both LMD & RMD.
Some drew node nodes at diff Levels to show order, that is fine as well.

—— 4 marks

—— 3 marks , 3 marks ——

LMD.

$S \Rightarrow \underline{A}aAb$
$\Rightarrow (\underline{b} \underline{AB}) aAb$
$\Rightarrow b (a\underline{A}b) \underline{B}aAb$

$\overset{*}{\Rightarrow} bad\,babaadb \overset{*}{\Longleftarrow}$
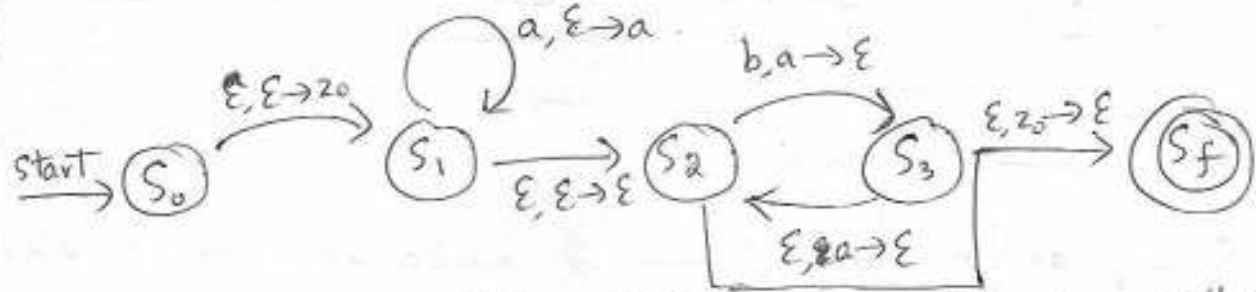
RMD.

$S \Rightarrow Aa\underline{A}b$
$\Rightarrow \underline{A}a (d) b$
$\Rightarrow (bA\underline{B}) a\underline{A}db$

4)



States and transitions of the PDA:
- start → $S_0$
- $S_0 \xrightarrow{\varepsilon, \varepsilon \to z_0} S_1$
- $S_1$ self-loop: $a, \varepsilon \to a$
- $S_1 \xrightarrow{\varepsilon, \varepsilon \to \varepsilon} S_2$
- $S_2 \xrightarrow{b, a \to \varepsilon} S_3$
- $S_3 \xrightarrow{\varepsilon, a \to \varepsilon} S_2$
- $S_3 \xrightarrow{\varepsilon, z_0 \to \varepsilon} S_f$

There are multiple solutions for this, can either push only once for every 2 'a', etc. Transition diagram or fn

——— 5 marks

ID :-

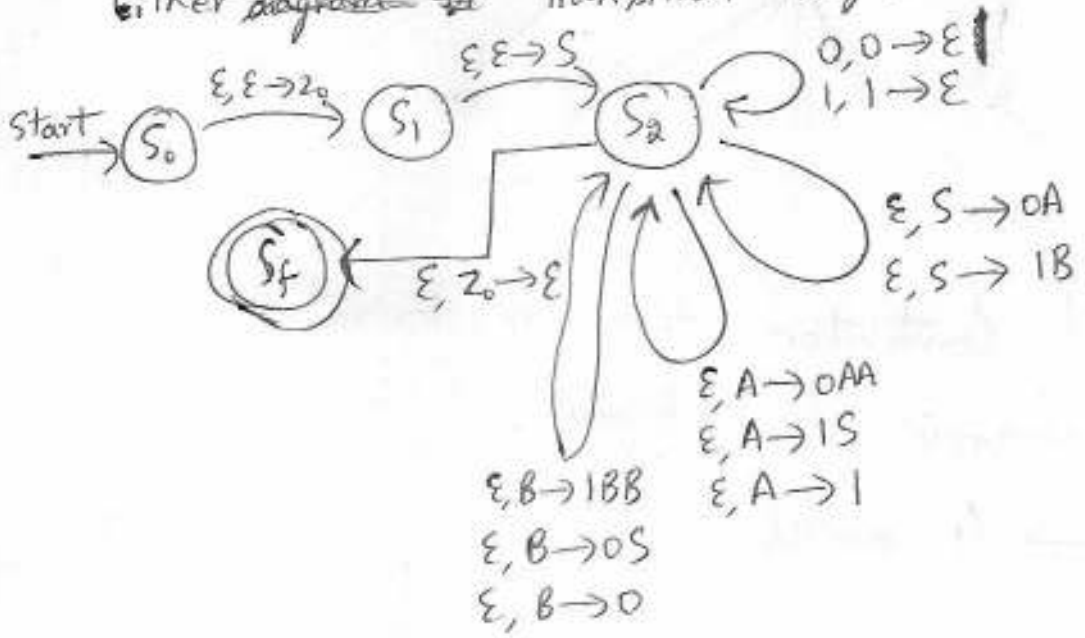① $(S_0, aaaabb, \varepsilon) \vdash (S_1, aaaabb, z_0)$
$\vdash (S_1, aaabb, az_0)$
$\vdash (S_1, aabb, aaz_0)$
$\vdash (S_1, abb, aaaz_0)$
$\vdash (S_1, bb, aaaaz_0)$
$\vdash (S_2, bb, aaaaz_0)$
$\vdash (S_3, b, aaaz_0)$
$\vdash (S_2, b, aaz_0)$
$\vdash (S_3, \varepsilon, az_0)$
$\vdash (S_2, \varepsilon, z_0)$
$\vdash (S_f, \varepsilon, \varepsilon) \,//$

——— 5 marks.

ID must be on the PDA constructed.

**5)** a) Diagram + Explaination — 5 marks.

b) There are multiple constructions possible.
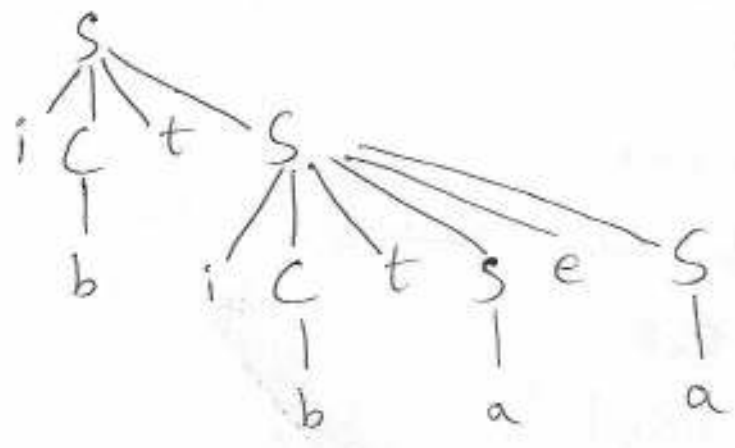Either diagram or transition diagram or fn.



$$\delta(S_2, \varepsilon, S) = \{(S_2, 0A), (S_2, 1B)\}$$

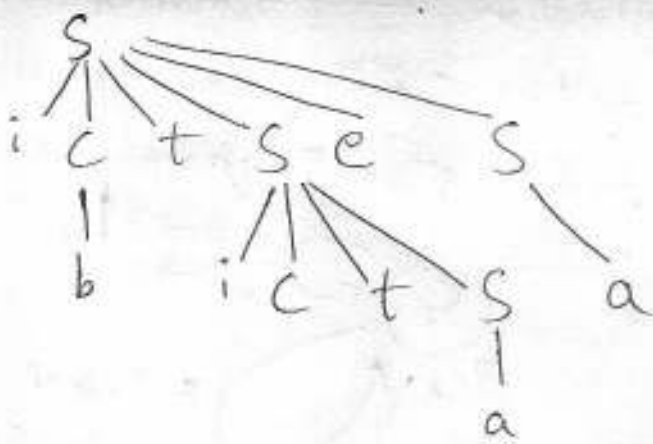$$\delta(S_2, \varepsilon, A) = \{(S_2, 0AA), (S_2, 1S), (S_2, 1)\}$$

$$\delta(S_2, \varepsilon, B) = \{(S_2, 1BB), (S_2, 0S), (S_2, 0)\}$$

—— 5 marks.

**6)** a)



1st construction, tree or, derivation.

The tree structure (derivation tree):
```
            S
   / / | \  \  \
  i c t S  e  S
  |   / | \      \
  b  i c t S a
           |
           a
```

2<sup>nd</sup> Construction, tree or derivation.

∴ Grammar is ambigious.

——— 4 marks.

b) Eliminate useless symbols first.

C useless because no termination.

Remove C.

Now A useless, because $S \to ABC$ got removed because of C. Now, no path from S to A.

∴ Grammar becomes :-

$$S \to BaB$$
$$B \to bBb \mid a \mid D$$
$$D \to \varepsilon.$$

——— 3 marks.

Now eliminate rule $D \to \varepsilon$.

∴ $S \to BaB$
$B \to bBb \mid a \mid \varepsilon$

Eliminate rule $B \to \varepsilon$

∴ $S \to BaB \mid aB \mid Ba \mid a$
$B \to bBb \mid a \mid bb$ //

——— 3 marks.

7) CNF defination + Explain
   —— 2 marks.

$S \longrightarrow aAa \mid AB$

~~Start~~ $A \longrightarrow BS \mid aBa \mid \varepsilon$

$B \longrightarrow aB \mid \varepsilon$

$\downarrow$

Fix
Start

$S_0 \longrightarrow S$
$S \longrightarrow aAa \mid AB$
$A \longrightarrow BS \mid aBa \mid \varepsilon$
$B \longrightarrow aB \mid \varepsilon$

$\xrightarrow{\text{Fix term.}}$

$S_0 \longrightarrow S$
$V_a \longrightarrow a$
$S \longrightarrow V_a A V_a \mid AB$
$A \longrightarrow BS \mid V_a B V_a \mid \varepsilon$
$B \longrightarrow V_a B \mid \varepsilon$

$\downarrow$ Fix ε productions n-t

$S_0 \longrightarrow S$
$V_a \longrightarrow a$
$S \longrightarrow V_a X_1 \mid AB$
$X_1 \longrightarrow A V_a$
$A \longrightarrow BS \mid V_a X_2 \mid \varepsilon$
$X_2 \longrightarrow B V_a$
$B \longrightarrow V_a B \mid \varepsilon$

$\xleftarrow{\text{Fix } \varepsilon\text{-prod.}}$

$S_0 \longrightarrow S$
$V_a \longrightarrow a$
$S \longrightarrow V_a X_1 \mid AB\cancel{|}$
$X_1 \longrightarrow A V_a$
$A \longrightarrow BS \mid$
$\qquad V_a X_2 \cancel{|}$
$X_2 \longrightarrow B V_a$
$B \longrightarrow V_a B$
$S \longrightarrow A \mid B \mid \varepsilon$
$X_1 \longrightarrow V_a$
$A \longrightarrow S$
$X_2 \longrightarrow V_a$
$B \longrightarrow V_a$

$\therefore S_0 \longrightarrow S \mid \varepsilon$
$V_a \longrightarrow a$
$S \longrightarrow V_a X_1 \mid AB \mid A \mid B$
$X_1 \longrightarrow A V_a \mid a$
$A \longrightarrow BS \mid V_a X_2 \mid S$
$X_2 \longrightarrow B V_a \mid a$
$B \longrightarrow V_a B \mid a$

$\xrightarrow{\text{fix unit-prod.}}$

$S_0 \longrightarrow S \mid \varepsilon$
$V_a \longrightarrow a$
$S \longrightarrow V_a X_1 \mid AB \mid$
$\qquad BS \mid V_a X_2 \mid$
$\qquad V_a B \mid a$
$X_1 \longrightarrow A V_a \mid a$
$A \longrightarrow BS \mid V_a X_2 \mid$
$\qquad V_a X_1 \mid AB \mid$
$\qquad BS \mid V_a X_2 \mid$
$\qquad V_a B \mid a$
$X_2 \longrightarrow B V_a \mid a$
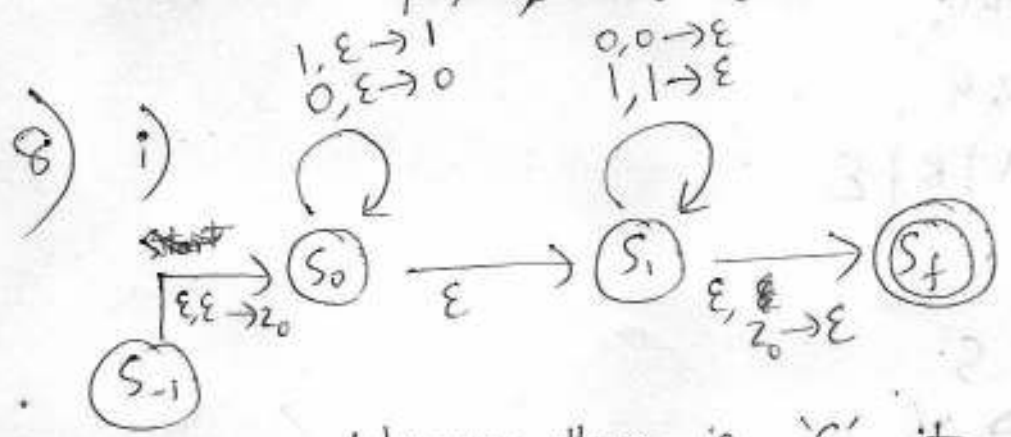$B \longrightarrow V_a B \mid a$ //

———7 marks.

Very few have got it correct to the end.
Full marks if all 5 requirements have been attempted.

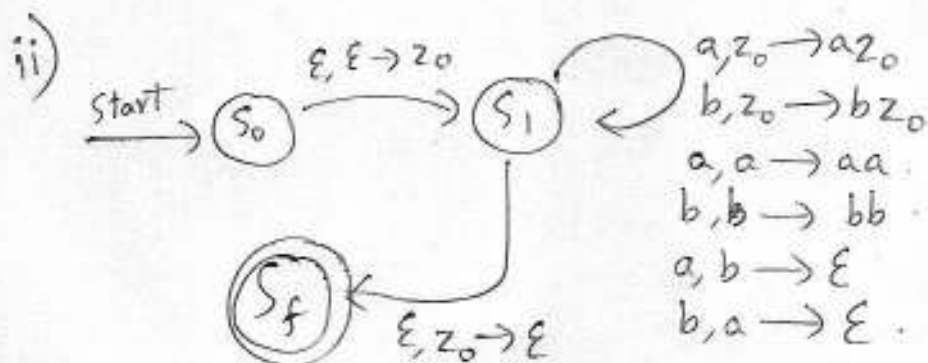There is dependency cycle. $S \longrightarrow A \longrightarrow S$
But, S must be constructed first. $\therefore$ Fix
~~$A \longrightarrow A$ first, then $A \longrightarrow S$.~~ $S \longrightarrow A$ first.



8) i)

$1, \varepsilon \to 1$
$0, \varepsilon \to 0$
$0, 0 \to \varepsilon$
$1, 1 \to \varepsilon$

Start $\xrightarrow{}$ $(S_0)$ $\xrightarrow{\varepsilon}$ $(S_1)$ $\xrightarrow[\varepsilon, Z_0 \to \varepsilon]{\varepsilon, \&}$ $((S_f))$
$\varepsilon, \varepsilon \to Z_0$
$(S_{-1})$

Whenever, there is '$\varepsilon$', it is non-deterministic.

Further, (i) cannot be solved by a D-PDA.

ii)

Start $\rightarrow$ $S_0$ $\xrightarrow{\varepsilon, \varepsilon \rightarrow z_0}$ $S_1$

$a, z_0 \rightarrow a z_0$
$b, z_0 \rightarrow b z_0$
$a, a \rightarrow a a$
$b, b \rightarrow b b$
$a, b \rightarrow \varepsilon$
$b, a \rightarrow \varepsilon$

$S_f$ $\xleftarrow{\varepsilon, z_0 \rightarrow \varepsilon}$

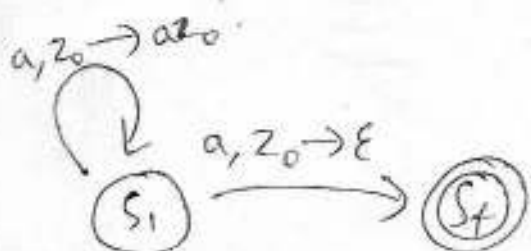The above is non-deterministic as it uses '$\varepsilon$'.
A deterministic version is:- This cannot
Assuming $z_0$ is already in stack. be solved by
a D-PDA as
we need to
check if stack
is empty at end
of input.

Start $\rightarrow$ $S_1$
$a, z_0 \rightarrow a z_0$
$b, z_0 \rightarrow b z_0$

$S_1$ $\xrightarrow{\varepsilon, z_0 \rightarrow \varepsilon}$ $S_f$
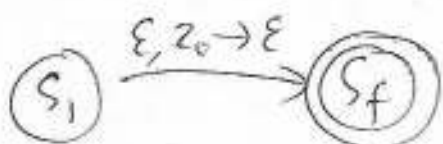
this is non-det., as '$\varepsilon$' is being used.

$a, z_0 \rightarrow a z_0$

$S_1$ $\xrightarrow{a, z_0 \rightarrow \varepsilon}$ $S_f$

this is also non-det, as there is already a
rule with transition
$$\delta(S_1, a, z_0) = (S_1, a z_0)$$
$\therefore$ Choice $\Rightarrow$ non-det.