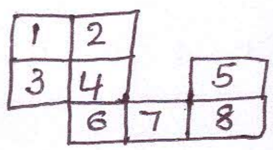

CMR
INSTITUTE OF
TECHNOLOGY

Improvement Test

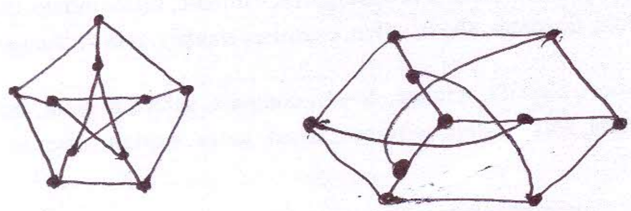
Sub:	DISCRETE MATHEMATICAL STRUCTURE					Code:	15CS36
Date:	19 / 11 / 2016	Duration:	90 mins	Max Marks:	50	Sem: III	Branch: CS, IS

Question 1 is compulsory and answer any six from questions 2 to 9.

		Marks	OBE CO	OBE RBT
1	Draw the Hasse diagram for the relation R defined by aRb iff a divides b, on the set of positive divisors of 36. Write the relation R.	[08]	CO3	L1
2	Find the rook polynomial for the board shown below:	[07]	CO6	L3



		Marks	CO	RBT
3	Determine the order of the graph in the following cases: (i) G is a cubic graph of 9 edges. (ii) G is regular with 15 edges. (iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.	[07]	CO5	L3
4	Write a short note on Konigsberg Bridge Problem.	[07]	CO5	L2
5	Prove that the following graphs are Isomorphic.	[07]	CO5	L1



		Marks	CO	RBT
6	Obtain the optimal prefix code for the message ROAD IS GOOD. Indicate the code for the following message.	[07]	CO5	L3
7	If a tree T has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4 and one vertex of degree 5, find the number of leaves in T.	[07]	CO5	L3
8	Prove that a tree with n vertices has n-1 edges.	[07]	CO5	L1
9	There are n pairs of children's gloves in a box. Each pair is of a different colour. Suppose the right gloves are distributed at random to n children, and thereafter the left gloves are also distributed to them at random. Find the probability that (i) no child gets a matching pair (ii) every child gets a matching pair (iii) exactly one child gets a matching pair.		CO2	L3

	Course Outcomes	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Verify the correctness of an argument using propositional and predicate logic and truth tables.	3	2	2	1	1	1	0	1	1	1	0	1
CO2:	Solve problems using counting techniques and combinatorics in the context of discrete probability.	3	2	2	1	1	1	0	1	1	1	0	1
CO3:	Solve problems involving relations and functions and their properties.	3	2	2	1	1	1	0	1	1	1	0	1
CO4:	Construct proofs using direct proof, proof by contradiction, proof by contradiction, and proof by cases and Mathematical induction.	3	2	2	1	1	1	0	1	1	1	0	1
CO5:	Explain and differentiate graphs and trees.	3	2	2	1	1	1	0	1	1	1	0	1
CO6:	Solve problems involving recurrence relations and generating functions.	3	2	2	1	1	1	0	1	1	1	0	1

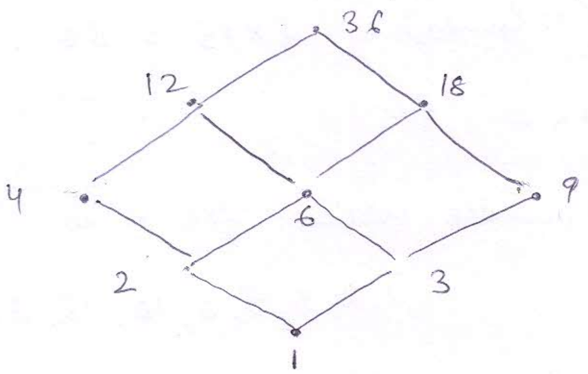
Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - Engineering knowledge; PO2 - Problem analysis; PO3 - Design/development of solutions; PO4 - Conduct investigations of complex problems; PO5 - Modern tool usage; PO6 - The Engineer and society; PO7- Environment and sustainability; PO8 – Ethics; PO9 - Individual and team work; PO10 - Communication; PO11 - Project management and finance; PO12 - Life-long learning

Solution - Improvement test
(CS & IS)

(1) $D_{36} = \{1,2,3,4,6,9,12,18,36\}$ — (1)

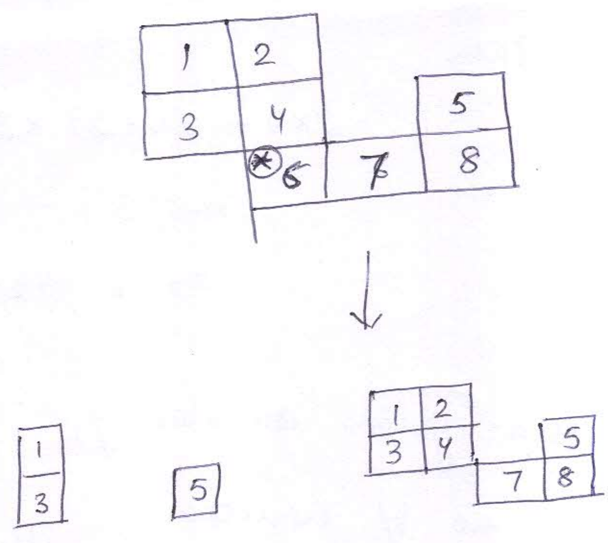
$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,12), (1,18), (1,36), (2,2)$
$(2,4), (2,6), (2,12), (2,18), (2,36)\ (3,6), (3,3), (3,12), (3,18)$
$(3,36), (4,4), (4,12), (4,36), (6,6), (6,12), (6,18), (6,36)$
$(12,12)\ (12,36), (18,18), (18,36), (36,36)\}\ (9,9), (9,18), (9,36)\}$ — (2)



— (5)

(2)

$r(D,x)$
$= 1 + 3x + x^2$ — (2)



D

E

$r(E,x)$

Here $r_1 = 7$ — (1)

$r_2 = 13$ — (1)

(1,4) (1,5), (1,7), (1,8) (2,3)
(2,5), (2,7), (2,8), (3,7), (3,8)
(4,7), (4,8) (5,7)

$r_3 = 6$ — (1)

(1,4,7), (1,4,8), (2,3,7), (2,3,8), (1,5,7), (2,5,7)

$\therefore\ r(E,x) = 1 + 7x + 13x^2 + 6x^3$

By expansion formula — $r(C,x) = x\,r(D,x) + r(E,x)$ — (1)

$r(C,x) = x(1+3x+x^2) + (1+7x+13x^2+6x^3)$
$= 1 + 8x + 16x^2 + 7x^3$ — (1)

(3) (i) G is a cubic graph.

Let no. of vertices is n, then degree of each vertex is 3

& by Handshaking property — 1

$$3+3+ \dots n \text{ times} = 2 \times e = 2 \times 9 = 18$$

$$3n = 18 \Rightarrow n = 6 \quad \text{— 2}$$

(ii) Let the degree of each vertex is k

$$\therefore \quad k+k+ \dots (n \text{ times}) = 2 \times 15 = 30$$

$$nk = 30$$

$k = \frac{30}{n}$, possible values for n is

1, 2, 3, 5, 6, 10, 15, 30 — 2

(iii) Let n be the no. of vertices

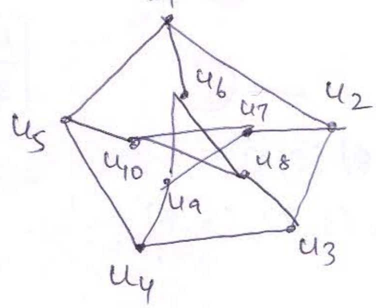
Then

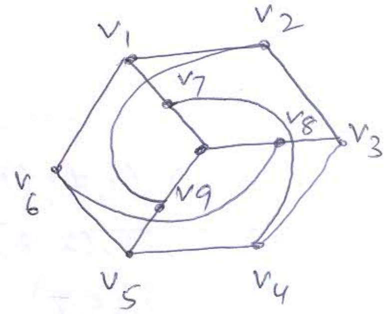
$$2 \times 4 + (n-2) \times 3 = 2 \times 10 = 20$$

$$(n-2)3 = 20 - 8 = 12 \quad \text{— 2}$$

$$3n = 12 + 6 = 18 \Rightarrow n = 6$$

(5) Both graphs are having same no of vertices i.e. $n = 10$ — 1





Both the graphs are having same no. of edges i.e. $e = 15$ — 1

& Every vertex is of degree 3 — 1

Consider 1-1 correspondence b/w the vertices

$u_i \leftrightarrow v_i$, when $i = 1, 2, 3 \dots 10$ — 1

This gives 1-1 correspondence b/w the edges so — (2)
that the adjacency of the vertices is preserved.

— (1)

∴ They are isomorphic.

(6) ROAD IS GOOD

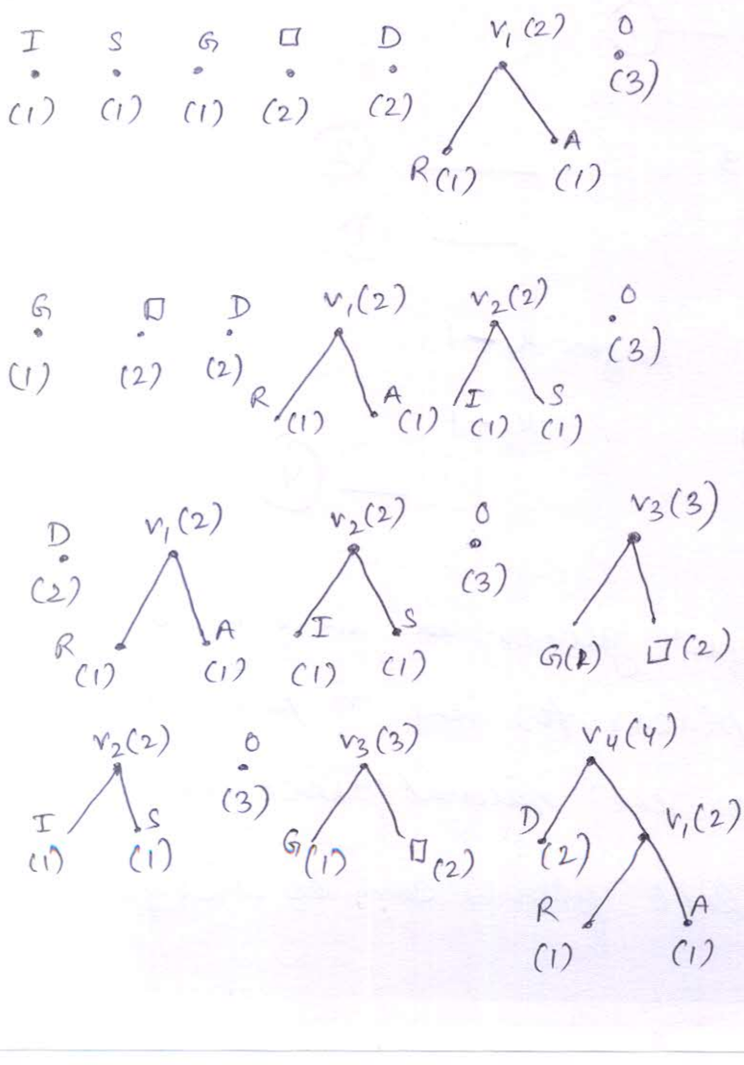
R : 1	I : 1
O : 3	S : 1
A : 1	G : 1
D : 2	□ : 2

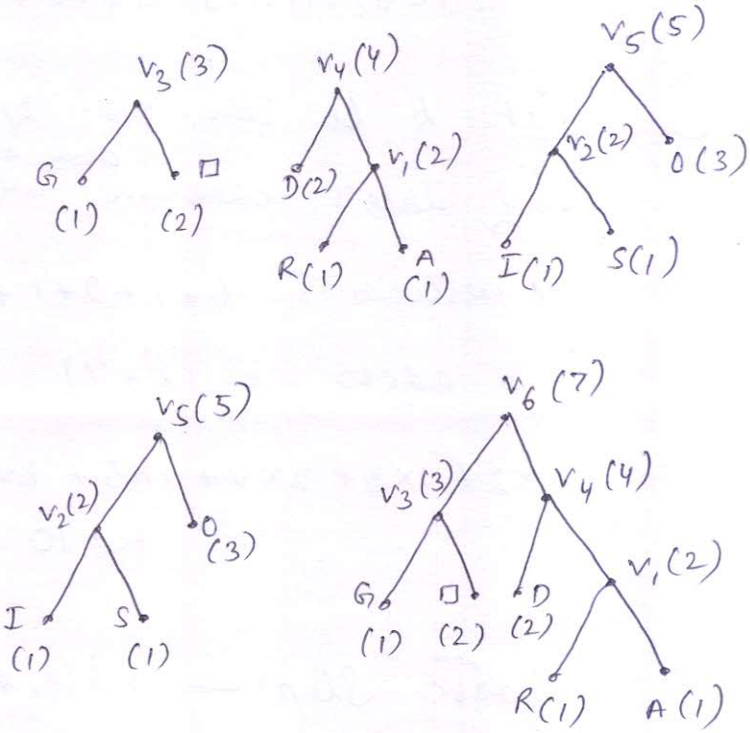
— (1)

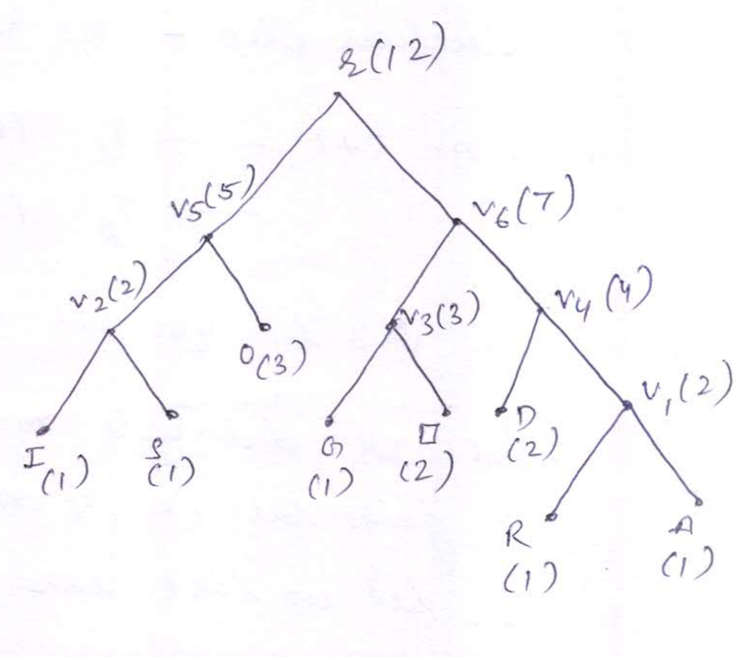
First we arrange the letters & □ in non-decreasing order of their weight

R	A	I	S	G	□	D	O
(1)	(1)	(1)	(1)	(1)	(2)	(2)	(3)

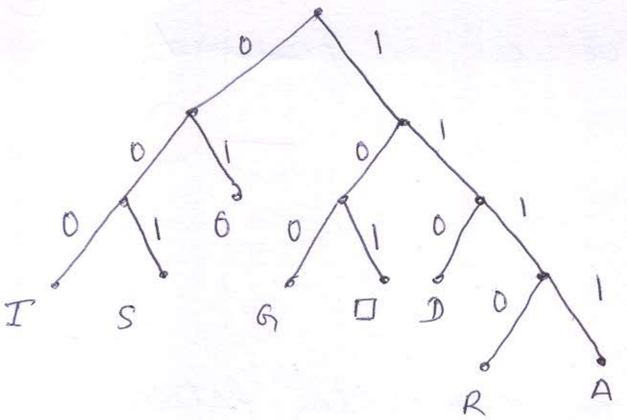
Now we construct an optimal tree having these symbols as leaves by Huffman's procedure







— (4)



R : 1110   G : 100
A : 1111   □ : 101
I : 000   D : 110
S : 001   O : 01

— (1)

∴ Code for the given message is

1110 01 111 1110 101 000 00110 100 010 1110 — (1)

(7) Let k be the no. of leaves then by — (1)
using Tree's theorem and hand shaking prop. we have —

No of vertices $= 4+1+2+1+k = k+8$ — (1)

∴ No of edges $= (k+7)$ — (1)

$\Rightarrow \quad 4\times2 + 1\times3 + 2\times4 + 1\times5 + k\times1 = 2(k+7)$ — (2)

$k = 10$ — (1)

(8) Basic Step — $n = 1, 2, 3$ — (2)

Induction step — $n = k$ — (1)

for $n = k+1$ ⟶ T_1 (k_1) edges $k_1 - 1$
⟶ T_2 (k_2) " $k_2 - 1$ — (4)

∴ It's true for all n.

(9) First we distribute n right gloves in any order that gives the set of n places for the n pairs of gloves. Let us take these as the natural place for the pairs of gloves. Now the left gloves can be distributed — (1)

in $n!$ ways

(i) No child getting a matching pair, it is derangement

The no. of derang. d_n

$$\text{Prob. } p = \frac{d_n}{n!}$$ — 2

(ii) Every child gets a matching pair, that is only one case — 2

$$\therefore p = \frac{1}{n!}$$

(iii) Exactly one child gets a matching pair

$$p = \frac{d_{n-1}}{n!}$$ — 2

4 Fig — 2

Short notes — 5