CMR INSTITUTE OF TECHNOLOGY

USN | | | | | | | | | |

Internal Assessment Test II – Nov. 2016

Sub:	**Modern Control Theory**	Code:	**10EE55**
Date: **02/11/2016**	Duration: **90 mins** Max Marks: **50** Sem: **IV**	Branch:	**EEE**

Note: Answer for FIFTY marks.

		Marks	CO	RBT
1.	Examine the complete state controllability and observability of a system with the following state-space model. $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-2 & -1 & -3\\ 0 & -2 & 1\\ -7 & -8 & -9\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}2\\ 1\\ 2\end{bmatrix}u, \quad y = \begin{bmatrix}4 & 6 & 8\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$	**[5+5]**	CO5	L3
2.	A single input system is given by the following state equation: $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\ 1 & -2 & 0\\ 2 & 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}10\\ 1\\ 0\end{bmatrix}u.$$ Design a state feedback controller which will enforce the closed-loop poles at $-1 \pm j2$, -6. Draw a block diagram of the closed-loop system.	**[8+2]**	CO6	L3
3.	Design a full state observer for the following system at eigen values $-2 \pm j2\sqrt{3}$ and -5. Draw a block diagram of the system with state observer. $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u, \quad y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$	**[8+2]**	CO6	L3
4.(a)	Describe PID Controller with block diagram.	**[4]**	CO1	L1
4.(b)	Draw the input-output characteristics of following nonlinearities and explain in detail: (i) Dead-zone (ii) Backlash.	**[6]**	CO3	L4
5.	Draw the phase plane trajectory for the following equation using isocline method: $$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0.$$ Given $\zeta = 0.5$, $\omega_n = 1$ rad/s, with $x = 0$ and $\dot{x} = 2$ as initial condition.	**[10]**	CO4	L3
6.	Construct the phase trajectory by delta method for a nonlinear system represented by differential equation $$\ddot{x} + 4\lvert\dot{x}\rvert\dot{x} + 4x = 0.$$ Choose initial conditions as $x(0) = 1$ and $\dot{x}(0) = 0$.	**[10]**	CO4	L3

(OBE: CO, RBT)

	Marks	OBE	
		CO	RBT
7. A second order system is described by $$\dot{x} = Ax, \text{ where } A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$$ Assuming matrix Q to be identity matrix, solve for P in the equation $A^{\mathrm{T}}P + PA = -Q$. Find Liapunov function $V(x)$. Use Liapunov theorem and examine the stability of the system at origin.	**[10]**	CO5	L5
8.(a) Examine the sign definiteness of the following quadratic functions: (i) $V(x) = -2x_1^2 - 2x_2^2 - 4x_3^2 - 2x_1x_2 + 4x_2x_3 + 4x_1x_3$ (ii) $V(x) = -2x_1^2 - x_2^2 - 4x_3^2 - 2x_1x_2 + 2x_2x_3 + 4x_1x_3.$	**[6]**	CO1	L1
8.(b) Explain Liapunov's theorems on (i) stability and (ii) asymptotic stability.	**[4]**	CO1	L1

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Explain state-space model of dynamic systems, sign definiteness of functions, Liapunov's stability criteria.	3	0	1	0	0	0	0	0	1	0	0	0
CO2:	Relate transfer function and state-space model for LTI systems.	3	0	0	0	0	0	0	0	1	0	0	0
CO3:	Identify elements with significant nonlinear behaviour in practical systems.	3	1	1	0	0	0	0	0	1	0	0	0
CO4:	Analyze solution of linear and non-linear state equations.	3	1	0	0	0	0	0	0	1	0	0	0
CO5:	Assess state controllability and observability of dynamic systems, stability of linear and non-linear systems using state-space model.	3	0	0	0	0	0	0	0	1	0	0	0
CO6:	Design state feedback controller and state observer.	3	0	2	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

10EE55 : Modern Control Theory - IAT-2.

Nov - 2016.

CMR

1. Examine the complete state controllability and observability of a system with the following state-space model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} u \ , \quad y = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} .$$

State Controllability : No. of States : $n = 3$.

$$Q_c = [B \quad AB \quad A^2B \cdots A^{n-1}B].$$

$$Q_c = \begin{bmatrix} 2 & -11 & 142 \\ 1 & 0 & -40 \\ 2 & -40 & 437 \end{bmatrix} \ ; \quad \text{Rank}\{Q_c\} = 3.$$

= No. of state Variables.

Hence by Kalman's test the given state-space model is complete state controllable.

State observability:
$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 4 & 6 & 8 \\ -64 & -80 & -78 \\ 674 & 848 & 814 \end{bmatrix} \ ; \quad \text{Rank}\{Q_o\} = 3.$$

= No. of State Variables.

Hence by Kalman's test the given state-space model is complete state observable.

2. A single input system is given by the following state equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 0 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 10 \\ 1 \\ 0 \end{bmatrix} u.$$

Design a state feedback controller which will enforce the closed-loop poles at $-1 \pm j2$, -6. Draw a block diagram of the closed-loop system.

Desired charc. eq: $\phi_{des}(s) = (s+1-j2)(s+1+j2)(s+6) = 0$.

$$(s^2+2s+5)(s+6) = 0$$

$$s^3+8s^2+17s+30 = 0.$$

Controllability matrix: $Q_c = [B \;\; AB \;\; A^2B \; \cdots \; A^{n-1}B]$; here $n=3$

$$Q_c = \begin{bmatrix} 10 & -10 & 10 \\ 1 & 8 & -26 \\ 0 & 21 & -75 \end{bmatrix}$$

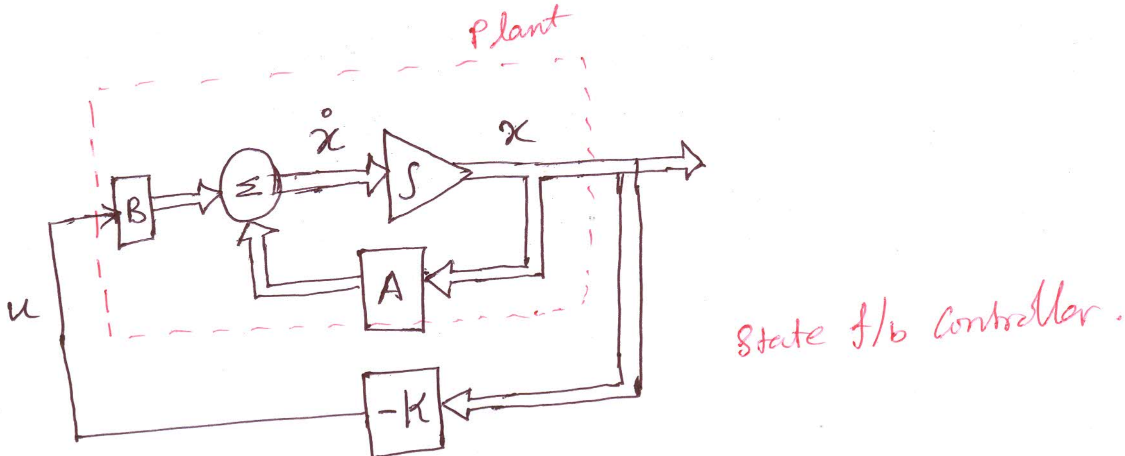
Ackermann's Formula for State-f/b. Controller pole-placement:

$$K = [0 \;\; 0 \cdots 1]\, Q_c^{-1}\, \phi_{des}(A).$$

$$= [0 \;\; 0 \;\; 1] \begin{bmatrix} 10 & -10 & 10 \\ 1 & 8 & -26 \\ 0 & 21 & -75 \end{bmatrix}^{-1} (A^3+8A^2+17A+30I)$$

$$= [-0.2222 \quad 4.2222 \quad -2]$$

Block diagram:



where $A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 0 \\ 2 & 1 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 1 \\ 0 \end{bmatrix}$,

$$K = \begin{bmatrix} -0.2222 & 4.2222 & -2 \end{bmatrix}$$

3. Design a full state observer for the following system at eigen values $-2 \pm j2\sqrt{3}$ and -5. Draw a block diagram of the system with state observer.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Desired ch. eq : $\phi_{o\_des}(s) = (s+2+j2\sqrt{3})(s+2-j2\sqrt{3})(s+5)$

$$\Rightarrow \quad = (s^2+4s+16)(s+5)$$

$$= s^3 + \cancel{7}\,9s^2 + \cancel{23}\,36s + \cancel{65}\,80$$

$$= s^3 + 9s^2 + 36s + 80$$

Observability Matrix:

Here $n = 3$:

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

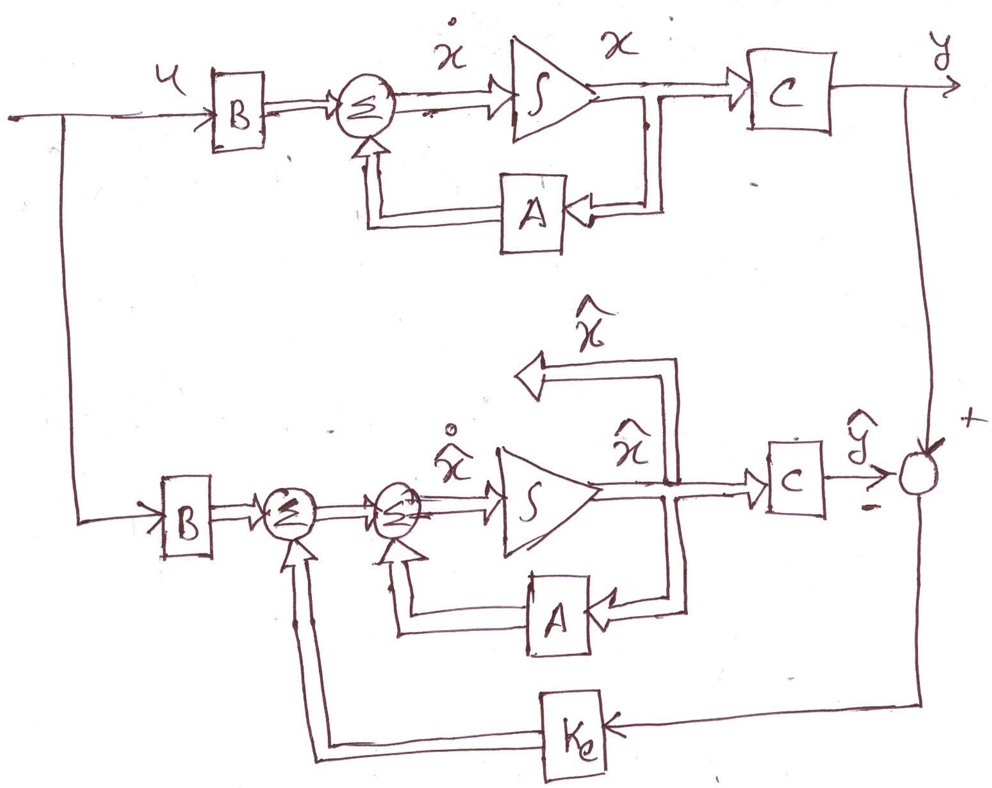
Ackermann's Formula for observer pole placement:

$$K_e = \phi_{o\_des}(A)\; Q_c^{-1} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

$$= \left(A^3 + \cancel{7}\,9A^2 + \cancel{23}\,36A + \cancel{65}\,80\right) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ \cancel{6}\,7 \\ -1\cancel{2} \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

Block diagram:



Q.7. A second order system is described by

$$\dot{x} = Ax, \text{ where } A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$$

Assuming matrix Q to be identity matrix, solve for P in the equation $A^T P + PA = -Q$. Find Liapunov function $V(x)$. Use Liapunov theorem ~~and~~ to examine the stability of the system at origin.

Let P be a symmetric matrix:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \quad ; \quad -Q = A^T P + PA$$

$\Rightarrow$

$$-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -P_{12} & -P_{22} \\ P_{11}-P_{12} & P_{12}-P_{22} \end{bmatrix} + \begin{bmatrix} -P_{12} & P_{11}-P_{12} \\ -P_{22} & P_{12}-P_{22} \end{bmatrix}$$

$$= \begin{bmatrix} -2P_{12} & P_{11}-P_{12}-P_{22} \\ P_{11}-P_{12}-P_{22} & 2(P_{12}-P_{22}) \end{bmatrix}$$

$$\Rightarrow \left.\begin{array}{l} -2P_{12} = -1 \\ P_{11}-P_{12}-P_{22} = 0 \\ 2(P_{12}-P_{22}) = -1 \end{array}\right\} \Rightarrow \begin{array}{l} P_{12} = 1/2 \\ P_{11} = 1.5 = 3/2 \\ P_{22} = 1 \end{array}$$

$$P = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1 \end{bmatrix} \quad ; \quad P_{11} > 0, \ |P| > 0.$$

P : Positive definite matrix.

Liapunov function: $V(x) = x^T P x$.

Positive definite function.

$$\|x\| \to \infty \Rightarrow V(x) \to \infty$$

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x}$$

$$= x^T A^T P x + x^T P A x$$

$$= x^T (A^T P + PA) x$$

$$= x^T (-Q) x.$$

$\dot{V}(x)$ is negative definite, hence by Liapunov's theorem the system is Asymptotic Stable in-the-large at origin.

8(a) Examine the sign definiteness of the following quadratic functions:

(i) $V(x) = -2x_1^2 - 2x_2^2 - 4x_3^2 - 2x_1x_2 + 4x_2x_3 + 4x_1x_3.$

(ii) $V(x) = -2x_1^2 - x_2^2 - 4x_3^2 - 2x_1x_2 + 2x_2x_3 + 4x_1x_3.$

(i) $V(x) = -2x_1^2 - 2x_2^2 - 4x_3^2 - 2x_1x_2 + 4x_2x_3 + 4x_1x_3.$

$$= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} -2 & -1 & 2 \\ -1 & -2 & 2 \\ 2 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

$$= x^T P x.$$

$$-P = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 4 \end{bmatrix}; \quad 2 > 0, \ \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} > 0, \ |-P| > 0.$$

$-P$ is Positive definite $\Rightarrow$ The given function is negative definite function.

(ii) $V(x) = -2x_1^2 - x_2^2 - 4x_3^2 - 2x_1x_2 + 2x_2x_3 + 4x_1x_3.$

$$= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} -2 & -1 & 2 \\ -1 & -1 & 1 \\ 2 & 1 & -4 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} . \qquad = x^T P x.$$

$$-P = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 1 & -1 \\ -2 & -1 & 4 \end{bmatrix} ; \qquad 2 > 0, \ \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} > 0, \ |-P| > 0.$$

$-P$ is Positive definite $\Rightarrow$ P is ~~posi~~ Negative definite, hence the given function is negative definite function.