CMR INSTITUTE OF TECHNOLOGY

Internal Assessment Test I – Sept. 2016

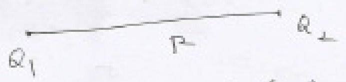
Sub:	Engineering Electromagnetics			**Code:**	15EC36
Date: 06/ 09 / 2016	Duration: 90 mins	Max Marks: 50	**Sem:** III	**Branch:**	EC/TC

Answer ALL questions choosing either (a) or (b) in each question.

Answer legibly and draw the diagrams neatly .Give proper units wherever necessary.

1.a)	State and explain Coulomb's law in vector form. Derive an expression for Electric Field Intensity at point Q(x_2, y_2, z_2) due to a point charge placed at P(x_1, y_1, z_1) in Cartesian co-ordinates.	10

Soln. The force b/w two very small charged objects separated in vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the dist. b/w them.



i.e. $F = \frac{k Q_1 Q_2}{R^2}$

Q_1 & Q_2 → +ve or -ve quantities of charge
unit Coulomb (C)

R → separation in m.

k → constant of proportionality.

$k = \frac{1}{4\pi\epsilon_0}$

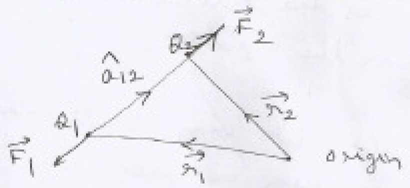
where, ϵ_0 → permittivity of free space.

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

$= \frac{1}{36\pi} \times 10^{-9}$ F/m.

F → Force in Newton.

1.(a) Vector form of Coulomb's law



$\vec{r}_1 \to$ locates Q_1

$\vec{r}_2 \to$ locates Q_2

Q_1, Q_2 of same sign, F_2 in the direction as indicated.

$F_2 \to$ force ~~ext~~ exerted on Q_2 by Q_1.

$\hat{a}_{12} \to$ unit vector along $\vec{R}_{12}$.

Then, the vector form of coulomb's law is

$$\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \hat{a}_{12}$$

where, $$\hat{a}_{12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$|\vec{R}_{12}| = R =$ Distance b/w the ~~tw~~ two charges.

Let, $\vec{F}_1$ be the force exerted by Q_1 on Q_2.

$\vec{r}_1 = \vec{r}_2 + \vec{R}_{21}$

$\Rightarrow \vec{R}_{21} = \vec{r}_1 - \vec{r}_2 = -(\vec{r}_2 - \vec{r}_1)$

$\therefore \hat{a}_{12} = -\hat{a}_{21}$.

	1.(a) $\therefore F_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2}(-\hat{a}_{21}) = -\vec{F_2}$, ⇒ Coulomb's law is a mutual force. Important observations: i) charges should be point charges and stationary in nature. ii) Should consider the signs of the charges to decide wheather the force will be attractive or repulsive. iii) Coulomb's law is linear. i.e. if $\vec{F_2} = -\vec{F_1}$ then, $n\vec{F_2} = -n\vec{F_1}$ where n is a scalar. iv) Force on a charge in the presence of several other charges is the sum of the forces on that charge due to each of the other charges acting alone.	
1.b)	Derive an expression for electric field intensity **E** at a distance ρ from an infinite line charge with uniform line charge density ρ_L C/m.	10

- consider the line charge to have a density of ρ_L C/m.

- let's assume a straight line charge extending along the z-axis in a cylindrical co-ordinate system from $-\infty$ to ∞.
- We desire the electric field intensity $\vec{E}$ at any point resulting from a uniform line charge density ρ_L.

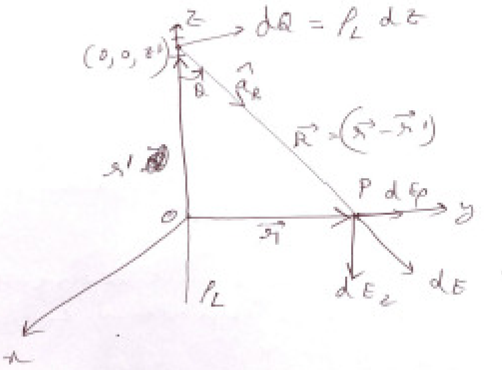
- How to decide on the symmetry:
  ① with which co-ordinates the field does not vary.
  ② which components of the field are not present.

Say we keep ρ and z constant and vary along ϕ.
- The line charge ~~ϕ~~ appears the same from every angle.

$\therefore$ Azimuthal symmetry is present and no field component may vary with ϕ.

If we vary z, line charge remains the same.
- But if we vary ρ, we get variation of field.

ϕ — no charge produces a ϕ component of electric intensity. $\boxed{E_\phi = 0}$



– consider a general point P(0, y, 0) on the y-axis to determine the field.

– we consider incremental charge,

$$dQ = \rho_L \, dz'.$$

– Aim to find incremental field.

$$\therefore d\vec{E} = \frac{\rho_L \, dz' \, (\vec{r} - \vec{r}\,')}{4\pi\epsilon_0 |r - r'|^3}.$$

where, $\vec{r} = y \, \hat{a}_y = \rho \, \hat{a}_\rho$

$\vec{r}\,' = z' \, \hat{a}_z$.

$\therefore \vec{r} - r' = (\rho \hat{a}_\rho - z' \hat{a}_z)$

$$\therefore d\vec{E} = \frac{\rho_L \, dz' \, (\rho \hat{a}_\rho - z' \hat{a}_z)}{4\pi\epsilon_0 \, (\rho^2 + z'^2)^{3/2}}$$

we know only ρ component is present.

$$dE_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho \, dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

$$E_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho \, dz'}{4\pi\epsilon_0(\rho^2+z'^2)^{3/2}}$$

Let, $z' = \rho\cot\theta$

$\therefore dz' = -\rho\,\mathrm{cosec}^2\theta\, d\theta$

$z' = \rho \frac{\cos\theta}{\sin\theta}$

$\infty = \frac{\cos 0}{\sin 0}$ $\Rightarrow$ for $z' = \infty$, $\theta = 0°$

$z' = -\frac{\cos(\pi)}{\sin(\pi)}$ $\Rightarrow$ $z' = -\infty$, $\theta = \pi$

$$E_\rho = \frac{\rho_L}{4\pi\epsilon_0}\int_{\pi}^{0} \frac{\rho . (-\rho\,\mathrm{cosec}^2\theta\, d\theta)}{(\rho^2+\rho^2\cot^2\theta)^{3/2}}$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\int_{\pi}^{0} \frac{-\rho^2\,\mathrm{cosec}^2\theta\, d\theta}{(\rho^2)^{3/2}(1+\cot^2\theta)^{3/2}}$$

$$= \frac{\rho_L}{4\pi\epsilon_0\rho}\int_{\pi}^{0} \frac{-\mathrm{cosec}^2\theta\, d\theta}{\mathrm{cosec}^3\theta}$$

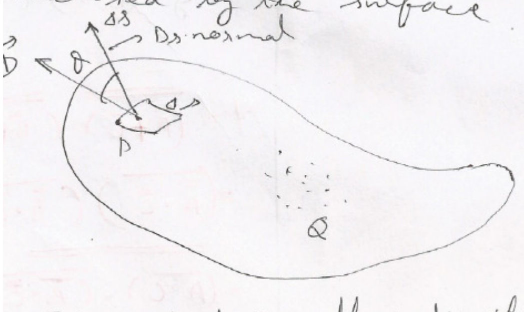
$$= \frac{\rho_L}{4\pi\epsilon_0\rho}\int_{\pi}^{0} -\sin\theta\, d\theta = \frac{\rho_L}{4\pi\epsilon_0\rho}\left[+\cos\theta\right]_{\pi}^{0}$$

$$= \frac{\rho_L}{4\pi\epsilon_0\rho}\left[1+1\right] = \frac{2\rho_L}{4\pi\epsilon_0\rho}$$

$$= \frac{\rho_L}{2\pi\epsilon_0\rho}$$

2.a)	Derive the mathematical form of Gauss's law and derive Gauss's Divergence theorem.	10

The electric flux passing through any closed surface is equal to the total charge enclosed by the surface



The electric flux density D_s at P due to charge Q. The total flux passing through ΔS is $D_s . \Delta S$.

cloud of pt. charge surrounded by a closed surface of any shape.

- If total charge Q, Q c flux passes through it

~~∴ At~~ +ve

$D_s \rightarrow$ electric flux value at one pt. in surface, varies from one pt. to the next

consider a small incremental area ΔS,

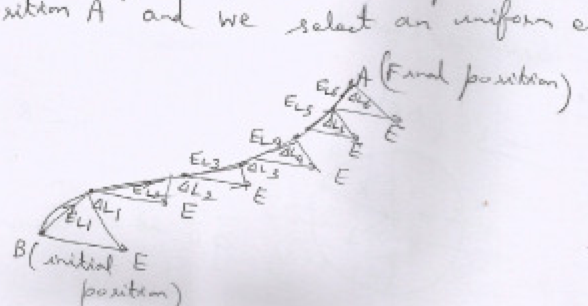
- vector quantity, with direction
- +ve for outward for closed surface.

$$\Delta\Psi = \text{flux crossing } \Delta S = D_{s,norm}\,\Delta S$$
$$= D_s \cos\theta\, \Delta S = \vec{D_s}\cdot\Delta\vec{S}$$
$$\therefore \Psi = \int d\Psi = \oint \vec{D_s}\cdot d\vec{S} = Q$$

(or)

2.b) Derive an expression for the work done in moving a point charge Q in the presence of an electric field **E**. **10**

We choose path from initial position B to final position A and we select an uniform electric field.



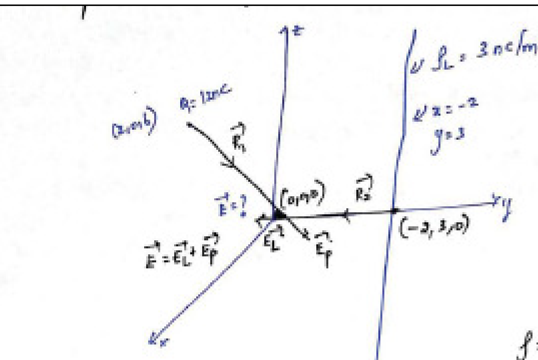
The path is divided into six segments $\Delta L_1, \Delta L_2 \ldots \Delta L_6$, and the components of $\vec{E}$ along each segment are denoted by $E_{L1}, E_{L2}, \ldots E_{L6}$. The work involved in moving a charge Q from B to A is then approximately,

$$W = -Q\,(E_{L1}\Delta L_1 + E_{L2}\Delta L_2 + \ldots E_{L6}\Delta L_6)$$
$$\Rightarrow W = -Q\,(\vec{E_1}\cdot\Delta\vec{L_1} + \vec{E_2}\cdot\Delta\vec{L_2} + \ldots \vec{E_6}\cdot\Delta \vec{L_6})$$

As we have assumed a uniform field,

$$\vec{E_1} = \vec{E_2} = \ldots = \vec{E_6}$$
$$W = -Q\vec{E}\cdot(\Delta L_1 + \Delta L_2 + \ldots \Delta L_6)$$
$$W = -Q\vec{E}\cdot\vec{L}_{BA}$$
$$W = -Q\int^{A} \vec{E}\cdot d\vec{L}$$

3.a)	Calculate the work done in moving a 2 μC charge from A (2, 1, -1) to B (8, 2, 1) in electric field **E** = y $\mathbf{a_x}$+ x $\mathbf{a_y}$ along hyperbola $x = \frac{8}{7-3y}$. The hyperbola, $x = \frac{8}{(7-3y)}$ $\quad$ $7x - 3xy = 8 \Rightarrow 7x - 8 = 3xy \Rightarrow \left(\frac{7}{3} - \frac{8}{3x}\right)$ $W = -2\times10^{-6}\left[\int_2^8\left(\frac{7}{3} - \frac{8}{3x}\right)dx + \int_1^2 \frac{8}{(7-3y)}dy\right]$ $= -2\times10^{-6}\left[\frac{7}{3}(8-2) - \frac{8}{3}(\ln x)_2^8 + \frac{8}{3}\ln\left(\frac{4}{1}\right)\right]$ Let, $7 - 3y = z$ $\Rightarrow -3dy = dz \Rightarrow dy = \frac{dz}{-3}$ when $y = 1,\ z = 4$ $y = 2,\ z = 1$ $\int_1^2 \frac{8}{7-3y}dy = \frac{8}{(-3)}\int_4^1 \frac{dz}{z}$ $= \frac{8}{3}\int_1^4 \frac{dz}{z} = \frac{8}{3}\ln\left(\frac{4}{1}\right)$ $\therefore W = -2\times10^{-6}\left[\frac{7}{\cancel{3}}\times\overset{2}{\cancel{6}} - \frac{8}{3}\ln\left(\frac{4}{1}\right) + \frac{8}{3}\ln\left(\frac{4}{1}\right)\right]$ $= -28\ \mu J$	10
3.b)	Find **E** at origin due to a point charge 12nC at (2, 0, 6) and a uniform line charge 3nC/m at x = -2, y = 3.	10



$\vec{R_1} = -2\vec{a_x} - 6\vec{a_z}$

$|\vec{R_1}| = \sqrt{4+36} = \sqrt{40}$

$\rho = |\vec{R_2}|$

$\vec{a_\rho} = \vec{a_{R_2}} = \dfrac{\vec{R_2}}{|\vec{R_2}|}$

$\vec{R_2} = 2\vec{a_x} - 3\vec{a_y}$

$|\vec{R_2}| = \sqrt{4+9} = \sqrt{13}$

$$\vec{E} = \vec{E_L} + \vec{E_p}$$

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho}\vec{a_\rho} + \frac{q}{4\pi\epsilon_0 |\vec{R_1}|^3}\vec{R_1}$$

$$= \frac{3\times10^{-9}}{2\pi \times \epsilon_0 \times \sqrt{13}} \times \frac{(2\vec{a_x} - 3\vec{a_y})}{\sqrt{13}} + \frac{12\times10^{-9}}{4\pi\times\epsilon_0} \times \frac{(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3\times10^{-9}\times 2\,(2\vec{a_x} - 3\vec{a_y})}{13} + \frac{12\times10^{-9}\,(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}\right]$$

$$= 9\times10^{9} \times 6\times10^{-9}\left[\frac{2}{13}\vec{a_x} - \frac{3}{13}\vec{a_y} - \frac{4}{(\sqrt{40})^3}\vec{a_x} - \frac{12}{(\sqrt{40})^3}\vec{a_z}\right]$$

$$= 54\left[0.138\,\vec{a_x} - 0.230\,\vec{a_y} - 0.0474\,\vec{a_z}\right]$$

$$\boxed{\vec{E} = 7.452\,\vec{a_x} - 12.42\,\vec{a_y} - 2.5596\,\vec{a_z}} \quad \text{V/m}$$

4.a)	Given potential V = $2x^2y$-5z. Compute V, **E**, **D**, ρ_v at a point P (-4, 3, 6). Indicate proper unit for each.	10

Given the potential field $V = 2x^2y - 5z$ and a point $P(-4, 3, 6)$ (37)

Find V at P, $\vec{E}$ at P, $\vec{a}_E$, $\vec{D}$ & ρ_v.

Potential at P = $V_p = 2(-4)^2(3) - 5(6) = 66\,V$

$$\vec{E} = -\nabla V = \frac{\partial V}{\partial x}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \frac{\partial V}{\partial z}\vec{a_z}.$$

$$= -\left[4xy\,\vec{a_x} + 2x^2\,\vec{a_y} - 5\,\vec{a_z}\right]$$

$$\boxed{\vec{E} = -4xy\,\vec{a_x} - 2x^2\vec{a_y} + 5\vec{a_z}}\ V/m$$

$$\vec{E}_p = (-4)(-4)(3)\,\vec{a_x} - 2(-4)^2\vec{a_y} + 5\vec{a_z}$$

$$\boxed{\vec{E}_p = 48\,\vec{a_x} - 32\,\vec{a_y} + 5\,\vec{a_z}}\ V/m$$

Direction of $\vec{E}$ $\vec{a_E}$ at p = $$\frac{48\vec{a_x} - 32\vec{a_y} + 5\vec{a_z}}{\sqrt{48^2 + 32^2 + 5^2}}$$

$$\boxed{\vec{a}_{E,P} = 0.829\,\vec{a_x} - 0.553\,\vec{a_y} + 0.0863\,\vec{a_z}}.$$

$$\vec{D} = \varepsilon\vec{E} = 8.854\times10^{-12} \times \left[(-4xy)\,\vec{a_x} - 2x^2\vec{a_y} + 5\vec{a_z}\right]$$

$$= \left[-35.4xy\,\vec{a_x} - 17.708x^2\,\vec{a_y} + 44.27\,\vec{a_z}\right]\times10^{-12}\ C/m^2$$

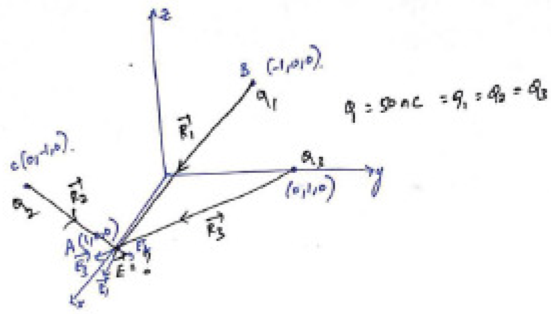
$$\boxed{\vec{D} = -35.4xy\,\vec{a_x} - 17.708x^2\vec{a_y} + 44.27\,\vec{a_z}}\ pC/m^2$$

$$\rho_v = \nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\boxed{\rho_v = -35.4y\ pC/m^3}$$

At P $\boxed{\rho_v = -106.2\ pC/m^3}$

4.b)	Calculate the electric field intensity **E** at A (1, 0, 0) due to three 50 nC point charges placed at B (-1, 0, 0), C (0, -1, 0) and D (0, 1, 0) respectively.	10

	 $\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3}$ $= \frac{q_1}{4\pi\epsilon_0 \lvert\vec{R_1}\rvert^3} \cdot \vec{R_1} + \frac{q_2}{4\pi\epsilon_0 \lvert\vec{R_2}\rvert^3} \cdot \vec{R_2} + \frac{q_3}{4\pi\epsilon_0 \lvert\vec{R_3}\rvert^3} \vec{R_3}$ $\vec{E} = 50\times10^{-9} \times 9\times10^{9} \left[\frac{\vec{R_1}}{\lvert\vec{R_1}\rvert^3} + \frac{\vec{R_2}}{\lvert\vec{R_2}\rvert^3} + \frac{\vec{R_3}}{\lvert\vec{R_3}\rvert^3} \right]$ $\vec{R_1} = 2\vec{a_x}$ $\quad \vec{R_2} = \vec{a_x} + \vec{a_y}$ $\quad \vec{R_3} = \vec{a_x} - \vec{a_y}$ $\lvert\vec{R_1}\rvert = 2$ $\quad \lvert\vec{R_2}\rvert = \sqrt{2}$ $\quad \lvert\vec{R_3}\rvert = \sqrt{2}$ $\vec{E} = 50\times9 \left[\frac{2\vec{a_x}}{2^3} + \frac{\vec{a_x}+\vec{a_y}}{(\sqrt{2})^3} + \frac{\vec{a_x}-\vec{a_y}}{(\sqrt{2})^3} \right]$ $= 450 \times \left[\frac{\vec{a_x}}{2^2} + \frac{\vec{a_x}}{(\sqrt{2})^3} + \frac{\vec{a_x}}{(\sqrt{2})^3} \right]$ $= 450 \left[\frac{1}{4} + \frac{1}{(\sqrt{2})^3} + \frac{1}{(\sqrt{2})^3} \right] \vec{a_x}$ $\boxed{\vec{E} = 430\ \vec{a_x}}$ V/m	
5.a)	Evaluate both sides of divergence theorem for the field $\mathbf{D} = 2xy\,\boldsymbol{a_x} + x^2\,\boldsymbol{a_y}$ C/m^3 and the rectangular parallelepiped formed by the planes x=0 and 1, y =0 and 2 and z =0 and 3.	10

Evaluate both sides of the divergence theorem for the field $\vec{D} = 2xy\,\hat{a}_x + x^2\hat{a}_y$ C/m^2 and the rectangular ~~pad~~ parallelepiped formed by the planes $x=0$ and 1, $y=0$ and 2, and $z=0$ and 3

Soln.

$$\oint \vec{D}\cdot d\vec{s} = \int_{vol} (\vec{\nabla}\cdot\vec{D})\, dv$$

$$\int_{front} \vec{D}\cdot d\vec{s} = \iint \left(2xy\,\hat{a}_x + x^2\hat{a}_y\right)\cdot dy\,dz\,\hat{a}_x$$

$$= \int_{y=0}^{2}\int_{z=0}^{3} 2xy\,dy\,dz = \int_{y=0}^{2}\int_{z=0}^{3} 2y\,dy\,dz$$

$$= 2\left[\frac{y^2}{2}\right]_0^2 \left[z\right]_0^3$$

$$= \cancel{4}\times 3 = \cancel{27}\,12$$

$$\int_{back} \vec{D}\cdot d\vec{s} = \int_{y=0}^{2}\int_{z=0}^{3}\left(2xy\,\hat{a}_x + x^2\hat{a}_y\right) dy\,dz\left(-\hat{a}_x\right)$$

$$= 0 \quad \left[\text{as on the back surface } x=0\right]$$

$$\int_{top} = \left(2xy\,\hat{a}_x + x^2\hat{a}_y\right)\cdot xy\,\hat{a}_z$$

$$= 0 = \int_{bottom}$$

$$\int_{left} \vec{D}\cdot d\vec{s} = \int_{x=0}^{1}\int_{z=0}^{3}\left(2xy\,\hat{a}_x + x^2\hat{a}_y\right) dx\,dz\left(-\hat{a}_y\right)$$

$$= \int_{x=0}^{1}\int_{z=0}^{3} - x^2\,dx\,dz = -\left[\frac{x^3}{3}\right]_0^1 \cdot 3$$

$$= -1$$

$$\int_{right} \vec{D}\cdot d\vec{s} = \int_{x=0}^{1}\int_{z=0}^{3} x^2\,dx\,dz = 1$$

$$\because \vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial x}(2xy) + \frac{\partial}{\partial y}(x^2)$$

$$= 2y$$

$$\therefore \int_{vol}(\vec{\nabla}\cdot\vec{D})\,dv = \int_{z=0}^{3}\int_{y=0}^{2}\int_{x=0}^{1} 2y\,dy\,dz\,dx$$

$$= 2\left[\frac{y^2}{2}\right]_0^2 \cdot 1 \cdot 3$$

$$= 12. \quad (\text{proved})$$

	(or)	
5.b)	Define current and current density and derive the equation of continuity of current.	10

Soln.

Current through a closed surface,

$$I = \oint_s \vec{J} \cdot \vec{ds}.$$

Outward flow of +ve charge is balance by a decrease of +ve charge within the closed surface.

Let, Q_i be the charge inside the closed surface

$$\therefore I = \oint_s \vec{J} \cdot \vec{ds} = -\frac{dQ_i}{dt}$$ → reduction in charge, giving -ve sign.

Using divergence theorem,

$$\oint_s \vec{J} \cdot \vec{ds} = \int_{vol} (\vec{\nabla} \cdot \vec{J})\, dv$$

Now, $Q = \int_{vol} \rho_v \, dv$

$$\therefore \int_{vol} (\vec{\nabla} \cdot \vec{J})\, dv = -\frac{d}{dt} \int_{vol} \rho_v \, dv$$

If the surface is constant derivative becomes a partial derivative.

$$\therefore \int_{vol} (\vec{\nabla} \cdot \vec{J})\, dv = \int_{vol} -\frac{\partial \rho_v}{\partial t}\, dv$$

This is true for any value however small.

This is true for an incremental volume.

$$\therefore (\vec{\nabla} \cdot \vec{J})\, \Delta v = -\frac{\partial \rho_v}{\partial t}\, \Delta v.$$

∴ Point form of continuity equation,

$$\boxed{(\vec{\nabla} \cdot \vec{J}) = -\frac{\partial \rho_v}{\partial t}}$$