CMR
INSTITUTE OF
TECHNOLOGY

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Internal Assesment Test - II

Sub:	Digital Image Processing							Code:	10EC763
Date:	4/11/2016	Duration:	90 mins	Max Marks:	50	Sem:	VII	Branch:	ECE(A,B,C,D) &TCE(A,B)

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1	Calculate Haar Transform for n=2 and discuss the properties of Haar Transform.	[10]	CO3	L3
2	Explain the following properties of 2D DFT. a) Translation b) Distributivity c) Separability d) Conjugate Symmetry	[10]	CO3	L2
3(a)	For the given orthogonal matrix A and image u, obtain the transformed image and basis images. $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $u = \begin{bmatrix} 1 & 2 \\ 8 & 4 \end{bmatrix}$	[4]	CO3	L3
(b)	Define slant transform. Make 4x4 slant transform matrix and show that it orthogonal.	[6]	CO3	L1
4	Define discrete cosine transform. Show that DCT over $NX1$can be computed using N-point FFT.	[10]	CO3	L1
5	Explain homomorphic filters in image enhancement with neat block diagram.	[10]	CO4	L4
6	Explain smoothing and sharpening filters in frequency domain.	[10]	CO4	L4
7	Discuss the characteristics of high boost filter for both frequency and spatial domain. Explain how high boost filtering enhances the image.	[10]	CO4	L4

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Describe the basic elements and applications of image processing, Analyze image sampling and quantization requirements and implications	3	-	-	-	-	-	1	-	-	-	-	-
CO2:	Apply Gray level transformations for Image enhancement, histogram equalization for image enhancement , Use and implement order-statistics image enhancement methods	3	-	-	-	-	-	1	-	-	-	-	-
CO3:	Acquire knowledge of solving problems related to different types of image transforms and applications.	3	-	-	-	-	-	1	-	-	-	-	-
CO4:	Design and implement two-dimensional spatial filters and frequency domain filter for image enhancement	3	-	-	-	-	-	1	-	-	-	-	-
CO5:	Model the image restoration problem in both time and frequency domains , Recognise and describe the image degradation models and restoration.	3	-	-	-	-	-	1	-	-	-	-	-
CO6:	Describe the representation of colours in digital colour images and basic concepts of colour image processing.	3	-	-	-	-	-	1	-	-	-	-	-

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

IAT-2.

Scheme & solutions

Digital Image Processing (10EC763)

1) $N = 2^n$, $0 \le k \le N-1$, $0 \le p \le n-1$

$p = 0 \Rightarrow q = 0, 1$

$p \ne 0 \Rightarrow 1 \le q \le 2^p$ $\qquad K = 2^p + q - 1$

$h_0(x) = h_{0,0}(x) = \frac{1}{\sqrt{N}}$

$$h_k(x) = h_{p,q}(x) = \frac{1}{\sqrt{N}} \begin{cases} 2^{p/2}, & \frac{q-1}{2^p} \le x < \frac{q-1/2}{2^p} \\ -2^{p/2}, & \frac{q-1/2}{2^p} \le x < q/2^p \\ 0, & o/w. \end{cases}$$

$x = \frac{m}{N}$ $\qquad 0 \le m \le N-1$ $\qquad x \in [0,1]$

$\Rightarrow x = 0, 1/N, 2/N, \ldots, \frac{N-1}{N}$.

> Let $n = 2 \Rightarrow N = 2^2 = 4$, $0 \le K \le 3$.

$0 \le p \le 1$

$p = 0 \Rightarrow q = 0, 1$

$p = 1 \Rightarrow 1 \le q \le 2 \Rightarrow q = 1, 2$.

K	0	1	2	3
p	0	0	1	1
q	0	1	1	2

1^{st} row $\rightarrow h_0(x) = \frac{1}{\sqrt{4}} = \frac{1}{2}$.

$$2^{nd} \text{ row} \rightarrow h_1(x) = h_{0,1}(x) = \frac{1}{2} \begin{cases} 1, & 0 \le x < 1/2 \\ -1, & 1/2 \le x < 1 \\ 0, & o/w. \end{cases}$$

$h_{0,1}(0/4) = h_{0,1}(0) = 1$
$h_{0,1}(1/4) = 1$
$h_{0,1}(2/4) = h_{0,1}(1/2) = -1$
$h_{0,1}(3/4) = -1$

$\left.\right\} \times 1/2$

3rd row $\longrightarrow h_2(x) = h_{1,1}(x) = \frac{1}{2}\begin{cases} \sqrt{2}, & 0 \le x < 1/4 \\ -\sqrt{2}, & 1/4 \le x < 1/2 \\ 0, & o/w \end{cases}$

$h_{1,1}(0) = \sqrt{2}$
$h_{1,1}(1/4) = -\sqrt{2}$
$h_{1,1}(2/4) = 0 = h_{1,1}(3/4)$

$\left.\right\} \times \frac{1}{2}$

4th row $\rightarrow h_3(x) = h_{1,2}(x) = \frac{1}{2}\begin{cases} \sqrt{2}, & 1/2 \le x < 3/4 \\ -\sqrt{2}, & 3/4 \le x < 1 \\ 0, & o/w \end{cases}$

$\therefore h_{1,2}(0) = 0 = h_{1,2}(1/4)$
$h_{1,2}(2/4) = \sqrt{2}$
$h_{1,2}(3/4) = -\sqrt{2}$

$\left.\right\} \times 1/2$

$$\Rightarrow H_r = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

~~Properties~~

2) (a) Translation

$$DFT\left[u(m-m_0, n-n_0)\right] = V(k,l)\, e^{-j\frac{2\pi}{N}(km_0+ln_0)}$$

where $V(k,l) = DFT\left[u(m,n)\right]$.

Proof

$$DFT\left[u(m-m_0, n-n_0)\right] = \sum_{m,n=0}^{N-1}\sum u(m-m_0, n-n_0) e^{-j\frac{2\pi}{N}(mk+ln)}$$

$$= \sum_{m,n=0}^{N-1}\sum u(m-m_0, n-n_0) e^{-j\frac{2\pi}{N}(m-m_0+m_0)k} e^{-j\frac{2\pi}{N}(n-n_0+n_0)l}$$

$$= e^{-j\frac{2\pi}{N}m_0k} e^{-j\frac{2\pi}{N}n_0l} \sum_{m,n=0}^{N-1}\sum u(m-m_0, n-n_0) e^{-j\frac{2\pi}{N}(m-m_0)k} e^{-j\frac{2\pi}{N}(n-n_0)l}$$

$$= e^{-j\frac{2\pi}{N}(m_0k+n_0l)} V(k,l)$$

(b) Distributivity

$$DFT\left[u_1(m,n) + u_2(m,n)\right] = DFT\left[u_1(m,n)\right] + DFT\left[u_2(m,n)\right].$$

Proof

$$DFT\left[u_1(m,n) + u_2(m,n)\right] = \sum_{m,n=0}^{N-1}\sum \left[u_1(m,n) + u_2(m,n)\right] e^{-j\frac{2\pi}{N}(mk+ln)}$$

$$= \sum_{m,n=0}^{N-1}\sum u_1(m,n) e^{-j\frac{2\pi}{N}(mk+ln)} + \sum_{m,n=0}^{N-1}\sum u_2(m,n) e^{-j\frac{2\pi}{N}(\ )}$$

$$= DFT\left[u_1(m,n)\right] + DFT\left[u_2(m,n)\right].$$

(c) Separability.

$$v(k,l) = \sum\sum_{m,n=0}^{N-1} u(m,n)\, e^{-j\frac{2\pi}{N}mk}\, e^{-j\frac{2\pi}{N}nl}$$

$$= \sum_{m=0}^{N-1}\left[\sum_{n=0}^{N-1} u(m,n)\, e^{-j\frac{2\pi}{N}nl}\right] e^{-j\frac{2\pi}{N}mk}$$

let $\sum_{n=0}^{N-1} u(m,n)\, e^{-j\frac{2\pi}{N}nl} = v(m,l)$

$$\Rightarrow v(k,l) = \sum_{m=0}^{N-1} v(m,l)\, e^{-j\frac{2\pi}{N}mk}$$

$\Rightarrow$ DFT of any dimension can be performed by applying a 1D transform on each dimension.

(d) Conjugate symmetry.

If DFT of $u(m,n) = v(k,l)$, then DFT of $u^*(m,n) \to v^*(-k,-l)$. Given $u(m,n)$ is real.

or $v(k,l) = v^*(-k,-l) = v^*(N-k, N-l)$

$0 \le k,l \le N-1$

Proof

$$v(k,l) = \sum\sum_{m,n=0}^{N-1} u(m,n)\, e^{-j\frac{2\pi}{N}mk}\, e^{-j\frac{2\pi}{N}nl}$$

$$v^*(k,l) = \sum\sum_{m,n=0}^{N-1} u(m,n)\, e^{j\frac{2\pi}{N}mk}\, e^{j\frac{2\pi}{N}nl}$$

$$\Rightarrow v(k,l) = v^*(-k,-l) = \underline{v^*(N-k, N-l)}$$

3(b) $N \times N$ slant transform is given by the recursion $\to$

(5)

$$S_n = \frac{1}{\sqrt{2}} \begin{bmatrix} \begin{matrix} 1 & 0 \\ a_n & b_n \end{matrix} & 0 & \begin{matrix} 1 & 0 \\ -a_n & b_n \end{matrix} & 0 \\ 0 & I_{N/2-2} & 0 & I_{N/2-2} \\ \begin{matrix} 0 & 1 \\ -b_n & a_n \end{matrix} & 0 & \begin{matrix} 0 & -1 \\ b_n & a_n \end{matrix} & 0 \\ 0 & I_{N/2-2} & 0 & -I_{N/2-2} \end{bmatrix} \begin{bmatrix} S_{n-1} & 0 \\ 0 & S_{n-1} \end{bmatrix}$$

$N = 2^n$, $I_M \rightarrow$ Identity matrix of $M \times M$.

$$S_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$b_n = (1 + 4a_{n-1}^2)^{-1/2}$$
$$a_n = 2b_n a_{n-1}$$, $a_1 = 1$.

Using these,
$$S_2 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3/\sqrt{5} & 1/\sqrt{5} & -1/\sqrt{5} & -3/\sqrt{5} \\ 1 & -1 & -1 & 1 \\ 1/\sqrt{5} & -3/\sqrt{5} & 3/\sqrt{5} & -1/\sqrt{5} \end{bmatrix}$$

$S^{-1} = S^T \Rightarrow$ orthogonal.

Here, $S_2 S_2^T = I$

4) $$c(k,n) = \begin{cases} \frac{1}{\sqrt{N}}, & k = 0, \quad 0 \le n \le N-1 \\ \sqrt{2/N} \cos \frac{\pi(2n+1)k}{2N}, & 1 \le k \le N-1, \\ & 0 \le n \le N-1 \end{cases}$$

1D DCT

$$v(k) = \alpha(k) \sum_{n=0}^{N-1} u(n) \cos\left[\frac{\pi(2n+1)k}{2N}\right]$$

$$\alpha(0) = 1/\sqrt{N}$$

$$\alpha(k) = \sqrt{2/N}, \quad 1 \le k \le N-1.$$

let $\left.\begin{array}{l} \tilde{u}(n) = u(2n) \\ \tilde{u}(N-n-1) = u(2n+1) \end{array}\right\} \quad 0 \le n \le \left(\frac{N}{2}\right) - 1$

$$v(k) = \alpha(k) \left\{ \left[\sum_{n=0}^{\frac{N}{2}-1} u(2n) \cos\left[\frac{\pi(4n+1)k}{2N}\right] + \sum_{n=0}^{\frac{N}{2}-1} u(2n+1) \cos\left[\frac{\pi(4n+3)k}{2N}\right] \right] \right\}$$

$$= \alpha(k) \left[\sum_{n=0}^{\frac{N}{2}-1} \tilde{u}(n) \cos\left[\frac{\pi(4n+1)k}{2N}\right] + \sum_{n=0}^{\frac{N}{2}-1} \tilde{u}(N-n-1) \cos\left[\frac{\pi(4n+3)k}{2N}\right] \right]$$

2nd term $\rightarrow$ let $n' = N-n-1 \Rightarrow n = N-n'-1$

$$\Rightarrow v(k) = \alpha(k) \sum_{n=0}^{\frac{N}{2}-1} \tilde{u}(n) \cos(\;) + \sum_{n'=N-1}^{N/2} \tilde{u}(n') \cos\left[\frac{\pi(4N-4n'-4+3)k}{2N}\right]$$

$$= \alpha(k) \sum_{n=0}^{\frac{N}{2}-1} \tilde{u}(n) \cos\left[\frac{\pi(4n+1)k}{2N}\right] + \sum_{n'=N/2}^{N-1} \tilde{u}(n') \cos\left[\pi\left(2 - \frac{4n'+1}{2N}\right)k\right]$$

$$= \alpha(k) \sum_{n=0}^{\frac{N}{2}-1} \tilde{u}(n) \cos\left[\frac{\pi(4n+1)k}{2N}\right] + \sum_{n'=N/2}^{N-1} \tilde{u}(n') \cos\left(\frac{4n'+1}{2N}\right)\pi k$$

$$= \alpha(k) \sum_{n=0}^{N-1} \tilde{u}(n) \cos\left[\frac{\pi(4n+1)k}{2N}\right]$$

$$= \mathrm{Re}\left[\alpha(k) e^{\frac{-j\pi k}{2N}} \sum_{n=0}^{N-1} \tilde{u}(n) e^{\frac{-j2\pi nk}{N}}\right]$$

$$= \mathrm{Re}\left[\alpha(k) W_{2N}^{k/2}\, \mathrm{DFT}[\tilde{u}(n)]\right]$$

3(a) $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $\quad u = \begin{bmatrix} 1 & 2 \\ 8 & 4 \end{bmatrix}$

$$V = AuA^T = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 8 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 9 & 6 \\ -7 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \underline{\underline{\frac{1}{2}\begin{bmatrix} 15 & 3 \\ -9 & -5 \end{bmatrix}}}$$

$$A^*_{0,0} = a^*_0 a^{*T}_0 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^*_{0,1} = a^*_0 a^{*T}_1 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$A^*_{1,0} = a^*_1 a^{*T}_0 = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$A^*_{1,1} = a^*_1 a^{*T}_1 = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Scheme IAT2
(10EC763)

1) Equations ——————————— (4M)
Matrix ——————————— (3M)
Properties ——————————— (3M)

2) Properties ——————————— (2.5 X 4M)

3(a) Transformed Image ——————————— (2M)
Basis Images ——————————— (2M)

3(b) Equations ——————————— (2M)
Matrix ——————————— (2M)
Orthogonality check ——————————— (2M)

4) Equations ——————————— (3M)
Derivation ——————————— (7M)

5) Block diagram ——————————— (4M)
Derivation ——————————— (6M)

6) Smoothing + Sharpening ——————————— (5+5M)

7) Frequency & Spatial domain ——————————— (5+5M)