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**Internal Assesment Test - II**

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| Sub: | ENGINEERING ELECTROMAGNETICS | | | | | | | Code: | 15EC36 |
| Date: | 02 / 11 / 2016 | Duration: | 90 mins | Max Marks: | 50 | Sem: | III | Branch: | ECE(A, B, C, D)/TCE |

Answer Any FIVE FULL Questions choosing either (a) or (b) from each question.

| | | Marks | OBE CO | OBE RBT |
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| 1(a) | Deduce Poisson's and Laplace's equations.<br>Check whether the following scalar fields are Laplacian:<br>i. $V = 2x^2 - 3y^2 + z^2$<br>ii. $V = (A\rho^4 + B\rho^{-4})\sin(4\phi)$ | [10] | CO1 | L1 |

Soln. From Gauss's law,

$\vec{\nabla}\cdot\vec{D} = \rho_v$

$\vec{D} = \epsilon\vec{E}$

and $\vec{E} = -\vec{\nabla}V$

$\therefore \vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot(\epsilon\vec{E}) = -\vec{\nabla}\cdot(\epsilon\vec{\nabla}V) = \rho_v$

$\Rightarrow -\vec{\nabla}\cdot(\vec{\nabla}V) = \frac{\rho_v}{\epsilon}$

$\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}V) = -\frac{\rho_v}{\epsilon}$

$\Rightarrow \nabla^2 V = -\frac{\rho_v}{\epsilon}$

$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$ → Poisson's equation.

Now, $\vec{\nabla}\cdot\vec{A} = \left(\hat{a}_x\frac{\partial}{\partial x} + \hat{a}_y\frac{\partial}{\partial y} + \hat{a}_z\frac{\partial}{\partial z}\right)\cdot(A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)$

$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\vec{\nabla}V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$

$\vec{\nabla}\cdot\vec{\nabla}V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$

$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}}$ → rectangular co-ordinates

If $\rho_v = 0$ then $\nabla^2 V = 0$

In cylindrical co-ordinates,

$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial\phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$

$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$

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| | i)<br>solving Laplace's equation<br>Soln $\frac{\partial V}{\partial x} = 4x$ \| $\frac{\partial V}{\partial y} = -6y$ \| $\frac{\partial V}{\partial z} = 2z$<br>$\frac{\partial^2 V}{\partial x^2} = 4$ \| $\frac{\partial^2 V}{\partial y^2} = -6$ \| $\frac{\partial^2 V}{\partial z^2} = 2$<br>$\therefore \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 4 - 6 + 2 = 0 = \nabla^2 V$<br>$\therefore \boxed{\nabla^2 V = 0}$ → given V satisfies Laplace's eqn.<br>ii)<br>(a) $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2}$<br>$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho\left(4A\rho^3 - 4B\rho^{-5}\right)\right]\sin 4\phi - \frac{1}{\rho^2}\frac{\partial}{\partial \phi}\left[\left(A\rho^4 + B\rho^{-4}\right) 4 \sin 4\phi\right]$<br>$= \frac{1}{\rho}\left(16A\rho^3 + 16B\rho^{-5}\right)\sin 4\phi - \frac{1}{\rho^2}\left(A\rho^4 + B\rho^{-4}\right) 16 \sin 4\phi$<br>$= 16A\rho^2 \sin 4\phi + 16B\rho^{-6}\sin 4\phi - 16A\rho^2 \sin 4\phi - 16B\rho^{-6}\sin 4\phi = 0$ | | | |
| | (OR) | | | |
| (b) | State and prove Uniqueness theorem.<br>"If the solution of Laplace's equation satisfies the boundary condition then that solution is unique by whatever method it is obtained."<br>"the solution of Laplace's equation gives the field which is unique, satisfying the same boundary conditions, in a given region<br>Proof:- $\nabla^2 V = 0$<br>we take two solution of Laplace's equation $V_1$ and $V_2$<br>$\nabla^2 V_1 = 0$ $\nabla^2 V_2 = 0$ ⇒ $\nabla^2 (V_1 - V_2) = 0$<br>we assume $V_1$ and $V_2$ are solutions of Laplace equation. of $V_1$ on boundary $V_{1b}$<br>— " — $V_2$ — " — $V_{2b}$.<br>$V_b$ → given potential value on boundary | [10] | CO1 | L1 |

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| | ∴ $V_{1b}=V_{2b}=V_b$<br>$\Rightarrow V_{1b}-V_{2b}=0$<br>considering the vector identity<br>$\vec{\nabla}\cdot(V\vec{D})=V(\vec{\nabla}\cdot\vec{D})+\vec{D}\cdot(\vec{\nabla}V)$<br>considering a scalar to be $(V_1-V_2)$<br>considering the vector to be $\vec{\nabla}(V_1-V_2)$<br>$\vec{\nabla}\cdot[(V_1-V_2)\vec{\nabla}(V_1-V_2)] = (V_1-V_2)[\vec{\nabla}\cdot\vec{\nabla}(V_1-V_2)] + [\vec{\nabla}(V_1-V_2)\cdot\vec{\nabla}(V_1-V_2)]$<br>taking volume integral on both sides<br>$\int_{vol}\vec{\nabla}\cdot[(V_1-V_2)\vec{\nabla}(V_1-V_2)]\,dv = \int_{vol}(V_1-V_2)\underbrace{\nabla^2(V_1-V_2)}_{0\ [\text{by hypothesis}]}\,dv + \int_{vol}[\vec{\nabla}(V_1-V_2)]^2\,dv$ (1)<br>$LHS=\oint_S[(V_1-V_2)\vec{\nabla}(V_1-V_2)]\cdot d\vec{S}$<br>$=\oint_S[(V_{1b}-V_{2b})\vec{\nabla}(V_{1b}-V_{2b})]\cdot d\vec{S}$<br>$=0$<br>∴ $\int_{vol}[\vec{\nabla}(V_1-V_2)]^2\,dv=0$ [Note, an integral may be zero if either the integrand is everywhere zero, or the integrand is +ve in some regions and -ve in other regions, and the contributions cancel algebraically. In this case the 1st reason holds good as $[\vec{\nabla}(V_1-V_2)]^2$ can not be -ve.<br>$\Rightarrow \vec{\nabla}(V_1-V_2)=0$<br>$V_1-V_2=$ constant.<br>on the boundary<br>$V_1-V_2=V_{1b}-V_{2b}=0$<br>$\Rightarrow V_1-V_2=0$<br>$\boxed{V_1=V_2}$ | | | |
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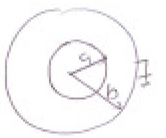
| 2(a) | Using Laplace's equation for potential field, derive the expression for capacitance of a cylindrical capacitor, where L is the length of the capacitor, a & b are inner and outer radii of the cylindrical layers and $V= V_0$ at $\rho = a$ and $V=0$ at $\rho = b$. | [10] | CO5 | L4 |



$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial V}{\partial\rho}\right]=0$$

$$\rho\frac{\partial V}{\partial\rho}=C_1$$

$$\frac{\partial V}{\partial\rho}=\frac{C_1}{\rho}$$

$$V=C_1\ln\rho+C_2$$

$V=0$ at $\rho=b$

$$0=C_1\ln b+C_2$$

$$V_0=C_1\ln a+C_2$$

$$-V_0=C_1[\ln b-\ln a]$$

$$V_0=C_1\ln(a/b)$$

$$\Rightarrow C_1=\frac{V_0}{\ln(a/b)}$$

$$C_2=\frac{-V_0\ln b}{\ln(a/b)}$$

$$V=\frac{V_0\ln\rho}{\ln(a/b)}-\frac{V_0\ln b}{\ln(a/b)}$$

$$\vec{E}=-\vec{\nabla}V$$

$$=\frac{-\partial V}{\partial r}\hat{a}_r$$

$$=\frac{\partial}{\partial r}\left[\frac{V_0\ln\rho}{\ln(a/b)}-\frac{V_0\ln(b)}{\ln(a/b)}\right]\hat{a}_\rho$$

$$\vec{E}=\frac{-V_0}{\rho\ln(a/b)}\hat{a}_\rho$$

$$\vec{D}=\epsilon_0\vec{E}$$

$$\vec{D}=\frac{-V_0\epsilon_0}{\rho\ln(a/b)}\hat{a}_\rho$$

$$\vec{D}=\frac{V_0\epsilon_0}{\rho\ln(b/a)}\hat{a}_\rho$$

$$Q=\vec{D}\cdot\vec{ds}$$

$$Q=\left[\frac{V_0\epsilon_0}{\rho\ln(b/a)}\hat{a}_\rho\right][2\pi\rho L\hat{a}_\rho]$$

$$Q=\frac{2\pi L V_0\epsilon_0}{\ln(b/a)}$$

$$C=\frac{Q}{V_0}$$

$$\boxed{C=\frac{2\pi L\epsilon_0}{\ln(b/a)}}$$

(OR)

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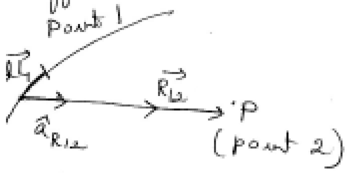
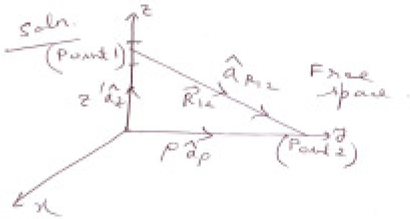
| (b) | State Biot-Savart law. Derive the expression for magnetic field intensity at a point due to an infinitely long conductor carrying current I Amperes. Consider ρ is the distance of the point from the conductor.<br>(a) Biot-Savart's law :-<br>Let's consider a differential current element as a vanishingly small section of a current-carrying filamentary conductor, where a filamentary conductor is the limiting case of a cylindrical conductor of circular cross-section as the radius approaches zero.<br>We assume a current I flowing in a differential vector length of the filament $d\vec{L}$. The Biot-Savart's law then states that, at any point P the mag. of the mag. field intensity produced by the differential element is proportional to the product of the current, the mag. of the differential length and the sine of the angle lying b/w the filament and a line connecting the filament to the point P at which the field is desired. Also, the magnitude of the mag. field intensity is inversely proportional to the square of the distance from the differential element to point P.<br><br><br>$d\vec{H_2} = \frac{I_1 d\vec{L_1} \times \hat{a}_{R_{12}}}{4\pi R_{12}^2}$<br>where $d\vec{H_2}$ → mag. field intensity produced by a differential current element $I_1 d\vec{L_1}$. The direction of $d\vec{H_2}$ is into the page. | [10] | CO1 | L1 |

Soln.




No variation with $z$ or $\phi$.

Point 2 chosen at $z=0$ plane.

$\therefore \vec{r} = \rho\hat{a}_\rho$ | $\vec{R}_{12} = \vec{r} - \vec{r}'$

$\vec{r}' = z'\hat{a}_z$ | $\vec{R}_{12} + z'\hat{a}_z = \rho\hat{a}_\rho$

$\Rightarrow \vec{R}_{12} = (\rho\hat{a}_\rho - z'\hat{a}_z)$

$$\therefore \hat{a}_{R_{12}} = \frac{(\rho\hat{a}_\rho - z'\hat{a}_z)}{\sqrt{\rho^2+z'^2}}$$

we take $\vec{dl} = dz'\hat{a}_z$

$$\therefore \vec{dH_2} = \frac{I\,dz'\hat{a}_z \times (\rho\hat{a}_\rho - z'\hat{a}_z)}{4\pi(\rho^2+z'^2)^{3/2}}$$

$$\therefore \vec{H_2} = \int_{-\infty}^{\infty} \frac{I\,dz'\,\hat{a}_z \times (\rho\hat{a}_\rho - z'\hat{a}_z)}{4\pi(\rho^2+z'^2)^{3/2}}$$

$$= \frac{I}{4\pi}\int_{-\infty}^{\infty}\frac{\rho\,dz'\,\hat{a}_\phi}{(\rho^2+z'^2)^{3/2}} = \frac{I\rho\hat{a}_\phi}{4\pi}\int_{-\infty}^{\infty}\frac{dz'}{(\rho^2+z'^2)^{3/2}}$$

let $z' = \rho\tan\theta$

$\Rightarrow dz' = \rho\sec^2\theta\,d\theta$

at, $z' = -\infty$, $\theta = -\pi/2$

$z' = +\infty$, $\theta = \pi/2$

$$\vec{H}_2 = \frac{I\rho\hat{a}_\phi}{4\pi}\int_{-\pi/2}^{\pi/2}\frac{\rho\sec^2\theta\,d\theta}{(\rho^2+\rho^2\tan^2\theta)^{3/2}}$$

$$= \frac{I\rho^2\hat{a}_\phi}{4\pi\rho^3}\int_{-\pi/2}^{\pi/2}\frac{\sec^2\theta}{\sec^3\theta}\,d\theta = \frac{I\hat{a}_\phi}{4\pi\rho}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

$$= \frac{I\hat{a}_\phi}{4\pi\rho}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} = \frac{I\hat{a}_\phi\,\cancel{2}}{\underset{2}{\cancel{4}}\pi\rho} = \frac{I}{2\pi\rho}\hat{a}_\phi \text{ A/m}$$

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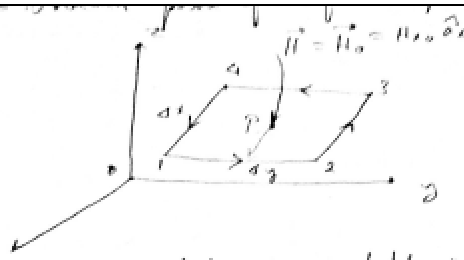
| 3(a) | State Ampere's circuital law. Derive the expression $\nabla \times \boldsymbol{H} = \boldsymbol{J}$ for static magnetic fields. | [10] | CO1 | L1 |

Ampere's circuital law states that,
The line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path,

$\therefore \oint \vec{H}\cdot\vec{dl} = I_{enclosed}$.

(c)




Incremental closed path in rectangular co-ordinates is selected for the application of Ampere's circuital law to determine the spatial rate of change of H.

— closed line integral of $\vec{H}$ about this path is approx. the sum of the four values of $\vec{H}\cdot\Delta\vec{L}$ on each side.

. current in $\hat{a}_z$ direction.

$$\therefore \left(\vec{H}\cdot\Delta\vec{L}\right)_{1-2} = H_{y,1-2}\,\Delta y$$

$$H_{y,1-2} = \left(H_{y_0} + \frac{\partial H_y}{\partial x}\cdot\frac{\Delta x}{2}\right)$$

$$\therefore \left(\vec{H}\cdot\Delta\vec{L}\right)_{1-2} = \left(H_{y0} + \frac{\partial H_y}{\partial x}\,\frac{\Delta x}{2}\right)\Delta y$$

$$\left(\vec{H}\cdot\Delta\vec{L}\right)_{2-3} = -\left(H_{x_0} + \frac{\partial H_x}{\partial y}\,\frac{\Delta y}{2}\right)\Delta x$$

[-ve sign because of path direction.]

For the path (3-4).

$$(\vec{H}\cdot\Delta\vec{l})_{3-4} = -\left(H_{y_0} + \frac{\partial H_y}{\partial x}\frac{\Delta x}{2}\right)\Delta y$$

going back along y-direction

path towards −ve x axis.

for the path (4-1),

$$(\vec{H}\cdot\Delta\vec{l})_{4-1} = \left(H_{x_0} - \frac{\partial H_x}{\partial y}\frac{\Delta y}{2}\right)\Delta x$$

[path in +ve x-direction]

$$\therefore \oint \vec{H}\cdot d\vec{l}$$

$$= H_{y_0}\Delta y + \frac{\partial H_y}{\partial x}\frac{\Delta x\Delta y}{2} - H_{x_0}\Delta x - \frac{\partial H_x}{\partial y}\frac{\Delta x\Delta y}{2}$$

$$- H_{y_0}\Delta y + \frac{\partial H_y}{\partial x}\frac{\Delta x\Delta y}{2} + H_{x_0}\Delta x - \frac{\partial H_x}{\partial y}\frac{\Delta x \Delta y}{2}$$

$$= \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\Delta x\Delta y = I_z = J_z\Delta x\Delta y$$

According to Ampere's circuital law,

$$\oint \vec{H}\cdot d\vec{l} = \Delta I = J_z\Delta x\Delta y.$$

$$\therefore \frac{\oint \vec{H}\cdot d\vec{l}}{\Delta x\Delta y} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = J_z$$

$$\text{as } \lim_{\Delta x\Delta y \to 0} \frac{\oint \vec{H}\cdot d\vec{l}}{\Delta x\Delta y} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = J_z \quad (1)$$

This gives a relation b/w line integral per unit area enclosed and the current per unit area enclosed i.e. current density.

Next take closed paths that are ⊥r to each of the remaining two co-ordinate axes.

Then similarly we get,

$$\lim_{\Delta y,\Delta z \to 0} \frac{\oint \vec{H}\cdot d\vec{l}}{\Delta y\Delta z} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x \quad (2)$$

$$\text{and} \lim_{\Delta z,\Delta x \to 0} \frac{\oint \vec{H}\cdot d\vec{l}}{\Delta z\Delta x} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y \quad (3)$$

(22)

$$\text{curl}\,\vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \vec{\nabla} \times \vec{H}$$

Formula of curl in other co-ordinate systems.

$$\vec{\nabla} \times \vec{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\hat{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\hat{a}_\phi + \left(\frac{1}{\rho}\frac{\partial (\rho H_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \phi}\right)\hat{a}_z$$

(cylindrical)

$$\vec{\nabla} \times \vec{H} = \frac{1}{r\sin\theta}\left(\frac{\partial (H_\phi \sin\theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right)\hat{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial}{\partial r}(r H_\phi)\right)\hat{a}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(r H_\theta) - \frac{\partial H_r}{\partial \theta}\right)\hat{a}_\phi$$

(spherical)

So $\oint \vec{E}\cdot d\vec{L} = 0$

$(\vec{\nabla} \times \vec{E}) = 0$

point form.

curl $\longrightarrow$ circulation per unit area.

$$\text{curl}\,\vec{H} = \vec{\nabla} \times \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{a}_z$$

$$= J_x\hat{a}_x + J_y\hat{a}_y + J_z\hat{a}_z = \vec{J}$$

... for same time ... ing conditions. $\boxed{(\vec{\nabla} \times \vec{H}) = \vec{J}} \longrightarrow$ Ampere's circuital law in point form

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| | (OR) | | | |
| (b) | Calculate the current density **J** at P (2, 3, 4). It is given that $\boldsymbol{H} = x^2 z\, \boldsymbol{a_y} - y^2 x\, \boldsymbol{a_z}$. | [10] | CO5 | L3 |

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| | $\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x^2 z & -y^2 x \end{vmatrix}$<br>$= \hat{a}_x \left[-2yx - x^2\right] + \hat{a}_y \left[+ y^2\right] + \hat{a}_z \left[2xz\right]$<br>$= \hat{a}_x (-2xy - x^2) + y^2 \hat{a}_y + 2xz\, \hat{a}_z$<br>At $P(2,3,4)$<br>$\vec{\nabla} \times \vec{H} = \hat{a}_x (-2.2.3 - 4) + 9\, \hat{a}_y + 2.2.4\, \hat{a}_z$<br>$= -16\, \hat{a}_x + 9\, \hat{a}_y + 16\, \hat{a}_z$<br>$\therefore \boxed{\vec{J} = -16\, \hat{a}_x + 9\, \hat{a}_y + 16\, \hat{a}_z \text{ A/m}}$ | | | |
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| 4(a) | Given $\boldsymbol{H} = 6xy\, \boldsymbol{a_x} - 3y^2\, \boldsymbol{a_y}$ A/m. Show that $\oint \boldsymbol{H}.\boldsymbol{dL} = \oiint (\boldsymbol{\nabla} \times \boldsymbol{H}).\boldsymbol{ds}$ for the rectangular path around the region $2 < x < 5, -1 < y < 1$ on the z = 0 plane. | [10] | CO5 | L3 |

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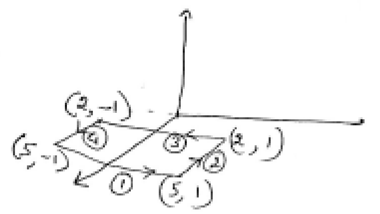
| | Soln.<br>$\vec{H}\cdot\vec{dl} = (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$<br>$= (6xy\,dx - 3y^2dy)$<br><br>Along path ①,<br>$\int \vec{H}\cdot\vec{dl} = \int_{y=-1}^{1}(6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\cdot dy\,\hat{a}_y$<br>$= -3\int_{-1}^{1} y^2dy = -\cancel{3}\left[\frac{y^3}{\cancel{3}}\right]_{-1}^{1} = -2$<br>Along path ②,<br>$\int \vec{H}\cdot\vec{dl} = \int_{x=5}^{2}(6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\cdot dx\,\hat{a}_x$<br>$= \int_{x=5}^{2} 6xy\,dx = \overset{3}{\cancel{6}}\cdot 1\cdot\left[\frac{x^2}{\cancel{2}}\right]_5^2 = 3\cdot(4-25) =$<br>Along path ③<br>$\int \vec{H}\cdot\vec{dl} = \int_{y=1}^{-1}(6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\cdot dy\,\hat{a}_y$<br>$= -3\int_{y=1}^{-1} y^2dy = -\cancel{3}\left[\frac{y^3}{\cancel{3}}\right]_1^{-1} = -\cancel{3}\left[\frac{-2}{\cancel{3}}\right] = 2$<br>Along path ④<br>$\int \vec{H}\cdot\vec{dl} = \int_{x=2}^{5} 6xy\,dx = 6\cdot(-1)\cdot\left[\frac{x^2}{2}\right]_2^5$<br>$= -6\cdot[25-4] = -63.$<br>$\therefore \oint \vec{H}\cdot\vec{dl} = -2-63+2-63 = -126.$<br>$\nabla\times\vec{H} = \begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 6xy & -3y^2 & 0\end{vmatrix} = -6x\,\hat{a}_z.$<br>$\therefore \iint(\nabla\times\vec{H})\cdot\vec{ds} = -\iint 6x\,\hat{a}_z\cdot dx\,dy\,\hat{a}_z$<br>$= -6\cdot\left[\frac{x^2}{2}\right]_2^5\left[y\right]_{-1}^{1}$<br>$= -\frac{6}{\cancel{2}}[25-4]\cdot\cancel{2} = -6\cdot 21 = -126.$<br>$\therefore \oint\vec{H}\cdot\vec{dl} = \iint(\nabla\times\vec{H})\cdot\vec{ds}$ (Proved) | | | |
| | (OR) | | | |

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| (b) | Derive the expression for the scalar and vector magnetic potentials.<br><br>Scalar magnetic potential :-<br>It is designated as $V_m$, and its -ve gradient gives the magnetic field intensity.<br>$\boxed{\therefore \vec{H} = -\vec{\nabla} V_m}$<br>This defn should not contradict our previous definitions.<br>$\rightarrow \vec{\nabla} \times \vec{H} = J = \vec{\nabla} \times (-\vec{\nabla} V_m) = 0$<br>curl of gradient of any scalar is zero.<br>$\therefore$ If $\vec{H}$ to be defined as the gradient of a scalar magnetic potential, then current density should be zero throughout the region in which the scalar magnetic potential to be defined.<br>i.e. $\boxed{\vec{H} = -\vec{\nabla} V_m \quad (\vec{J} = 0)}$<br>It also satisfies Laplace's eqn.<br>i.e. $\vec{\nabla} \cdot \vec{B} = \mu_0 \vec{\nabla} \cdot \vec{H} = 0$ (in free space)<br>$\therefore \mu_0 \vec{\nabla} \cdot (-\vec{\nabla} V_m) = 0$<br>$\therefore \nabla^2 V_m = 0 \quad (\vec{J} = 0)$<br>$V_m$ is not a single valued function of position<br>for co-axial cable $a < \rho < b$.<br>then, $\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi$.<br>$\vec{H} = -\vec{\nabla} V_m$<br>$\Rightarrow \frac{I}{2\pi\rho} = -\frac{1}{\rho} \frac{\partial V_m}{\partial \phi}$. | [10] | CO1 | L1 |

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| | $\therefore \frac{\partial V_m}{\partial \phi} = -\frac{I}{2\pi}$.<br>$\therefore V_m = -\frac{I}{2\pi}\phi$ [setting constant of integration = 0]<br>~~Now with~~ let, $V_m = 0$ at $\phi = 0$.<br>proceed counter-clockwise rotation, ~~again~~ $V_m = -I$.<br>Then after one complete rotation,<br>But we already said $V_m = 0$ at $\phi = 0$.<br>$\vec{\nabla} \times \vec{E} = 0$<br>$\oint \vec{E} \cdot d\vec{L} = 0$.<br>$V_{ab} = -\int_b^a \vec{E} \cdot d\vec{L}$ independent of the path.<br>But here, $\vec{\nabla} \times \vec{H} = 0$, when $\vec{J} = 0$<br>and $\oint \vec{H} \cdot d\vec{L} = I$, even if $\vec{J}$ is zero along the path of integration.<br>With every turn current increases by $I$.<br>$\therefore V_{m\,a,b} = -\int_b^a \vec{H} \cdot d\vec{L}$ (specified path).<br>Vector magnetic potential:-<br>$\vec{\nabla} \cdot \vec{B} = 0$ we known.<br>divergence of the curl of any vector = 0<br>$\therefore$ we chose $\vec{B} = (\vec{\nabla} \times \vec{A})$<br>(where, $\vec{A} \rightarrow$ magnetic vector potential<br>$\therefore \vec{H} = \frac{1}{\mu_0}(\vec{\nabla} \times \vec{A})$<br>$\therefore \vec{\nabla} \times \vec{H} = \vec{J} = \frac{1}{\mu_0}(\vec{\nabla} \times \vec{\nabla} \times A)$<br>$A = \int_{vol} \frac{\mu J\, dv}{4\pi R}$ $\boxed{\vec{A} = \oint \frac{\mu_0 I\, d\vec{L}}{4\pi R}}$ | | | |
| | | | | |
| 5(a) | Deduce Maxwell's equations in point form and integral form for time varying **E** and **H** fields. | [10] | CO1 | L1 |

Stationary path.

magnetic flux → time varying quantity.

$\therefore$ emf $= \oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$

Applying stokes's theorem

$$\int_s (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

Surface integrals taken over identical general surfaces.

$$\therefore (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

$$\Rightarrow \boxed{(\vec{\nabla}\times\vec{E}) = -\frac{\partial \vec{B}}{\partial t}} \rightarrow \text{differential or point fo.}$$

$$\boxed{\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial B}{\partial t}\cdot d\vec{s}}$$

From Ampere's circuital law, $(\vec{\nabla}\times\vec{H}) = \vec{J}$ ... (1)

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) \equiv 0 = \vec{\nabla}\cdot\vec{J}$

identically.

$\Rightarrow \vec{\nabla}\cdot\vec{J} = 0.$

But from eqn. of continuity.

$$\vec{\nabla}\vec{J} = -\frac{\partial \rho_v}{\partial t}$$

$\therefore$ For time varying field we add an unknown term $\vec{G}$ to eqn. (1)

$\vec{\nabla}\times\vec{H} = \vec{J} + \vec{G}$

$\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{G}$

$\Rightarrow \vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\vec{G}$

$\therefore \vec{\nabla}\cdot\vec{G} = \frac{\partial \rho_v}{\partial t}$

$\therefore \vec{\nabla}\cdot\vec{G} = \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) = \vec{\nabla}\cdot\frac{\partial \vec{D}}{\partial t}$

$\therefore \vec{G} = \frac{\partial \vec{D}}{\partial t}.$

$\therefore$ Ampere's circuital law in point form,

$$(\vec{\nabla}\times\vec{H}) = \left(\vec{J} + \frac{\partial D}{\partial t}\right)$$

| | | | | |
|---|---|---|---|---|
| | Faraday's law: $\oint \vec{E}\cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$<br>$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \int_s (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -$<br>$\therefore \boxed{\oint \vec{E}\cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}}$<br>Ampere's circuital law;<br>$\oint \vec{H}\cdot d\vec{l} = I + \int_s \frac{\partial \vec{D}}{\partial t}\cdot d\vec{s}$<br>$\int_s \vec{\nabla}\times\vec{H} = \int_s \vec{J}\cdot d\vec{s} + \int_s \frac{\partial \vec{D}}{\partial t}\cdot d\vec{s}$<br>$\Rightarrow \vec{\nabla}\times\vec{H} = I + \int_s \frac{\partial \vec{D}}{\partial t}\cdot d\vec{s}$ | | | |
| | (OR) | | | |
| (b) | Given $V = x(z-ct)$ and $\boldsymbol{A} = x(\frac{z}{c} - t)\boldsymbol{a_Z}$. Show $\boldsymbol{\nabla}.\boldsymbol{A} = -\mu_0\varepsilon_0\frac{\partial V}{\partial t}$ and $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$.<br>Calculate **E** and **D**.<br>Sln (a) $\vec{\nabla}\cdot\hat{A} = \frac{\partial A_z}{\partial z}$<br>$= \frac{\partial}{\partial z}\left[x\left(\left(\frac{z}{c}\right) - t\right)\right] = \frac{x}{c} = x\sqrt{\mu_0\epsilon_0}$<br>Now, $\frac{\partial V}{\partial t} = -cx = \frac{-x}{\sqrt{\mu_0\epsilon_0}}$<br>$\therefore -\mu_0\epsilon_0\frac{\partial V}{\partial t} = -\mu_0\epsilon_0\frac{(-x)}{\sqrt{\mu_0\epsilon_0}} = x\sqrt{\mu_0\epsilon_0}$<br>$\therefore \vec{\nabla}\cdot\hat{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$ for free space, implying the given statement will hold for media. | [10] | CO6 | L3 |

i) Now, $\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$

$$\vec{\nabla} V = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\left[x(z-ct)\right]$$

$$= (z-ct)\,\hat{a}_x + x\,\hat{a}_z$$

$$\frac{\partial \vec{A}}{\partial t} = \frac{\partial}{\partial t}\left[x\left\{\left(\frac{z}{c}\right)-t\right\}\right]\hat{a}_z = -x\,\hat{a}_z$$

$$\therefore \vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} = -(z-ct)\,\hat{a}_x - \cancel{x\hat{a}_z} + \cancel{x}$$

$$= (ct-z)\,\hat{a}_x \ \text{V/m}$$

$$\therefore \vec{D} = \epsilon_0 \vec{E} = \epsilon_0 (ct-z)\,\hat{a}_x \ \text{C/m}^2$$

| Course Outcomes | | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 | PO9 | PO10 | PO11 | PO12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CO1: | Explain and solve problems related to Basic Concepts of Electric Fields, Magnetic Fields and Electromagnetic Waves and basic Concepts to Solve Complex Problems in Electric Fields, Magnetic Fields and Electromagnetic Waves. | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| CO2: | Interpretation of Gradient, Divergence and Curl Operators. | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| CO3: | Interpretation of Maxwell’s Equations in differential and integral forms and interpretation of wave propagation in free space and dielectrics. | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| CO4: | Apply the knowledge gained in the design of Electric and Electronic Circuits, Electrical Machines and Antenna’s and Communication Systems. | 3 | 3 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| CO5: | Analyze different Charge and Current Configurations to derive Electromagnetic Field Equations. Analyze Poisson’s and Laplace’s Equations, Uniqueness theorem, and solution of Laplace’s equation. | 3 | 3 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| CO6: | Analyze Time-varying fields, Maxwell’s equations, wave propagation in free space and dielectrics. | 3 | 3 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |

| Cognitive level | KEYWORDS |
|---|---|

| | |
|---|---|
| L1 | List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc. |
| L2 | summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend |
| L3 | Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover. |
| L4 | Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer. |
| L5 | Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize. |

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*