CMR
INSTITUTE OF
TECHNOLOGY

USN | | | | | | | | | | |

IMPROVEMENT TEST

Sub:	ENGINEERING MATHEMATICS I							Code:	15MAT11
Date:	18/11/2016	Duration:	90 mins	Max Marks:	50	Sem:	I	Branch:	D,E,F,G,I,J,N

Answer All Questions

		Marks	OBE CO	OBE RBT
1	Expand $\log\,(1+Sin\,x)$ up to the term x^4 using Maclaurin's series.	[08]	CO1	L3
2	Find the angle of intersection between the curves $r = 6\,Cos\theta$ and $r = 2(1 + Cos\,\theta)$.	[06]	CO1	L3
3	Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form.	[06]	CO6	L3
4	A particle moves along the curve C: $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$ where t denotes time. Find the components of acceleration at $t = 2$ along the tangent and normal.	[06]	CO3	L3
5	Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both Solenoidal and Irrotational.	[06]	CO4	L3
6	Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is a conservative field. Find its scalar potential.	[06]	CO4	L3
7	Find the directional derivative of the function $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ along $2i - j - 2k$.	[06]	CO4	L3
8	Prove that $div\left(Curl\,\vec{A}\right) = 0$.	[06]	CO4	L3

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Apply nth derivate to obtain Taylor and Maclaurin's series of a given function. Evaluate the radius of curvature for cartesian, parametric polar and pedal equation.	2	3	-	-	-	-	-	1	1	1	-	-
CO2:	Apply partial derivatives to calculate rates of change of multivariate functions. Evaluate definite integrals.	2	3	-	-	-	-	-	1	1	1	-	-
CO3:	Analyze position, velocity, and acceleration in two or three dimensions using the calculus of vector valued functions.	2	3	-	-	-	-	-	1	1	1	-	-
CO4:	Evaluate curl and divergence of a vector valued functions which has various applications in electricity, magnetism and fluid flows.	2	3	-	-	-	-	-	1	1	1	-	-
CO5:	Solve first order ordinary Differential equations and model Newton's law of cooling.	2	3	-	-	-	-	-	1	1	1	-	-
CO6:	Evaluate matrices and determinants for solving systems of linear equations used in the different areas of Linear Algebra.	2	3	-	-	-	-	-	1	1	1	-	-

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

Improvement Test
Solution Manual

① $\log(1+\sin x)$

$$y(x) = y(0) + x y_1(0) + \frac{x^2}{2!} y_2(0) + \cdots \qquad -①$$

$y = \log(1+\sin x)$ $\qquad y(0) = 0$

$$y_1 = \frac{\cos x}{1+\sin x} \qquad y_1(0) = 1$$

$$y_2 = \frac{-\sin x - \cos x\, y_1}{1+\sin x} \qquad y_2(0) = -1 \qquad \Big\} -⑥$$

$$y_3 = \frac{-\cos x - 2y_2 \cos x + y_1 \sin x}{1+\sin x} \qquad y_3(0) = 1$$

$$y_4 = \frac{\sin x - 3\cos x\, y_3 + 3y_2 \sin x + y_1 \cos x}{1+\sin x} \qquad y_4(0) = -2$$

$$\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} \qquad — ①$$

② $r = 6\cos\theta$ $\qquad r = 2(1+\cos\theta)$

$\cot\phi_1 = -\tan\theta$ $\qquad \cot\phi_2 = -\tan\theta/2$ $\qquad$ — ①

$\phi_1 = \frac{\pi}{2} + \theta$ $\qquad \phi_2 = \frac{\pi}{2} + \frac{\theta}{2}$ $\qquad$ — ②

$$|\phi_1 - \phi_2| = \theta/2$$

$6\cos\theta = 2(1+\cos\theta)$ — (2)

$\theta = \frac{\pi}{3}$ — (1)

$\therefore \ |\phi_1 - \phi_2| = \frac{\pi}{6} = 30^\circ$.

(3) The symmetric matrix of the QF is

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$ — (4)

$\lambda_1 = 0$, $\lambda_2 = 3$, $\lambda_3 = 15$

Corresponding eigen vectors

$X_1 = [1, 2, 2]'$ $X_2 = [2, 1, -2]$ $X_3 = [2, -2, 1]'$

Canonical form is $\underline{3y_2^2 + 15y_3^2}$. — (2)

(4) $\vec{r} = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k$

$\vec{v} = \frac{d\vec{r}}{dt} = (3t^2 - 4)i + (2t - 4)j + (16t - 9t^2)k$

$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 6ti + 2j + (16 - 8t)k$

$(\vec{v})_{t=2} = 8i + 8j - 4k = \vec{V}$ — (2)

$(\vec{a})_{t=2} = 12i + 2j - 20k = \vec{A}$

along the tangent

$= \vec{A} \cdot \hat{n}$ where $\hat{n} = \frac{\vec{V}}{|\vec{V}|}$ — ②

$= 16$

Along the normal

$= |\vec{A} - (\vec{A} \cdot \hat{n})\hat{n}|$ — ②

$= 2\sqrt{73}.$

(5) $\vec{F} = \frac{xi + yj}{x^2 + y^2}$

$\text{div}\,\vec{F} = \nabla \cdot \vec{F}$

$= \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \left(\frac{x}{x^2+y^2} i + \frac{y}{x^2+y^2} j\right)$

$= \frac{1}{(x^2+y^2)^2} \left(y^2 - x^2 + x^2 - y^2 \right) = 0.$ — ③

$\therefore$ $\vec{F}$ is solenoidal

$\text{curl}\,\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{x^2+y^2} & \frac{y}{x^2+y^2} & 0 \end{vmatrix}$

$= k\left(\frac{-2xy}{(x^2+y^2)^2} + \frac{2xy}{(x^2+y^2)^2}\right) = \vec{0}$ — ③

$\therefore$ $\vec{F}$ is irrotational.

(6) we have to show that

$\text{curl}\,\vec{F} = \vec{0}$

$$\text{curl}\,\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y+z & z+x & x+y \end{vmatrix}$$

$$= \vec{0} \qquad — (2)$$

consider $\nabla \phi = \vec{F}$

$$\frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k = (y+z) i + (z+x) j + (x+y) k$$

$\phi = xy + xz + f_1(y, z)$

$\phi = yz + xy + f_2(x, z) \qquad — (3)$

$\phi = xz + yz + f_3(x, y)$

$\therefore \phi = xy + yz + zx. \qquad — (1)$

(7) $\phi = x^2yz + 4xz^2$

$$\nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$$

$— (2)$

$(\nabla \phi)_{(1, -2, -1)} = 8i - j - 10k$

unit vector in the direction of $2i - j - 2k$ is

$$\hat{n} = \frac{2i - j - 2k}{3} \qquad — (2)$$

∴ directional derivative

$$\nabla\phi\cdot\hat{n} = (8i - j - 10k)\,\frac{(2i - j - 2k)}{3}$$

$$\nabla\phi\cdot\hat{n} = \frac{37}{3}. \quad \text{— (2)}$$

(8) $\operatorname{div}(\operatorname{curl}\vec{A}) = 0$ or $\nabla\cdot(\nabla\times\vec{A}) = 0$ — (1)

$$\vec{A} = a_1 i + a_2 j + a_3 k$$

$$\operatorname{curl}\vec{A} = \nabla\times\vec{A} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_1 & a_2 & a_3 \end{vmatrix} \quad \text{— (1)}$$

$$= \sum i \left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) \quad \text{— (1)}$$

$$= \sum \frac{\partial}{\partial x} i \quad \sum i \left(\frac{\partial a_3}{\partial y} - \frac{\partial a_2}{\partial z}\right) \quad \text{— (2)}$$

$$= \sum \left(\frac{\partial^2 a_3}{\partial x\,\partial y} - \frac{\partial^2 a_2}{\partial x\,\partial z}\right) \quad \text{— (1)}$$

Thus $\operatorname{div}(\operatorname{curl}\vec{A}) = 0$, for any vector function $\vec{A}$