CMR
INSTITUTE OF
TECHNOLOGY

139

Internal Assessment Test 1 – September 2016

Sub:	Operations Research					Code:	10ME74
Date: 07/ 09 / 2016	Duration: 90 mins	Max Marks: 50		Sem:	7^{TH}	Branch:	ME

Note: Answer any five questions:

1a. A Firm makes two products X & Y And has a total production capacity of 9 ton's per day. X&Y Requiring the same production capacity the firm has a permanent contract to supply at least 2 ton's of X and at least 3 ton's of Y per day to another company each ton of X requires 20 Machine hours production time and each ton of Y requires 50 machine hours Production time the daily maximum possible no. of hours is 360 all the firms output can be Sold and the profit obtained is Rs 80 per ton of X and Rs120 per ton of Y respectively. Formulate The LPP and solve it graphically. (8 Marks)

1b. List the assumptions made in LPP. (2Marks)

2. Solve The LPP using Simplex Method. (10 Marks)
Minimize $Z = x_1 - 3x_2 + 2x3$ STC $3x_1 - x_2 + 3x3 \le 7$, $-2x_1 + 4x_2 \le 12$, $-4x_1 + 3x_2 + 8x3 \le 10$ $x_1, x_2, x3 \ge 0$.

3. Solve the LPP using Big-M-Method
$z_{MIN} = 4x_1 + x_2$ STC $3x_1 + x_2 = 3$, $4x_1 + 3x_2 >= 6$, $x_1 + 2x_2 \le 4$, $x_1, x_2 \ge 0$. (10 Marks)

4a. Explain slack variable, surplus variable, Artificial variable, Binding & Non- binding constraint. (5 Marks)

4b. List the principles of duality in LPP. (5 Marks)

5. Show that the primal and dual solutions have the same optimal solution and solution can be read from the simplex table of each other. (10 Marks)
$z_{Max} = 2x_1 + x_2$ STC $x_1 + 5x_2 \le 10$, $x_1 + 3x_2 \le 6, 2x1 + 2x2 <= 8$ $x_1, x_2, \ge 0$

6a. Solve the LPP Using dual simplex method. (6 Marks)
$z_{Min} = 2x_1 + 2x_2 + 4x_3$ STC $2x_1 + 3x_2 + 5x_3 \ge 2$, $3x_1 + x_2 + 7x_3 <= 3$ $x_1 + 4x_2 + 6x_3 <= 5 x_1, x_2, x3 \ge 0$

6b. Explain briefly in LPP Infeasible solution, unbounded solution, Alternate optimal solution, Degenerate solutions with example. (4 Marks)

7. Determine the IBFS of the Given Transportation problem using North West Corner rule, Row minima method, Column minima method and least cost method. (10 Marks)

From/To	A	B	C	D	Capacity
1	8	6	5	4	100
2	7	3	9	2	150
3	6	5	4	3	175
Demand	75	200	25	125	

8. A company has to distribute coal from mines to different consumers at least transportation cost as per the delivery promise given in the following data find the best distribution of coal.

Three mines are located at west Bengal, bihar, Orissa that produce 60,35 & 45 Ton's per week respectively. Five consumers A,B,C,D,E who require 22,45,20,18 & 35 Ton's per week respectively.

The transportation cost/ton is given below: Find the optimal schedule using Vogel's approximation Method and conduct the test of optimality by MODI method. (10 Marks)

	A	B	C	D	E
W.B	4	1	3	4	4
Bihar	2	3	2	2	3
Orissa	3	5	2	4	4

CI CCI HOD

(2) Converting Minimization case to Maximisation Case

$Max\ z = -x_1 + 3x_2 - 2x_3$

Converting the inequalities into Equations by adding slack variables S_1, S_2, S_3 resp.

$3x_1 - x_2 + 3x_3 + S_1 = 7$

$-2x_1 + 4x_2 + 0x_3 + S_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

New o.f

$Max\ z = -x_1 + 3x_2 - 2x_3 + 0S_1 + 0S_2 + 0S_3$ — 3 MARKS

2 iterations — 2+2 MARKS ⇒ 4 MARKS

optimal Table — 3 MARKS
& optimal Solution

	c_j	-1	3	-2	0	0	0		
C_B	basis	x_1	x_2	x_3	S_1	S_2	S_3	Constant RHS	MR
-1	x_1	1	0	6/5	2/5	1/10	0	4	
3	x_2	0	1	3/5	1/5	3/10	0	5	
0	S_3	0	0	11	1	-1/2	1	11	
	Z	0	0	13/5	1/5	4/5	0		

Since $Z \geq 0$ Solution is optimal

$x_1 = 4$

$x_2 = 5$

$x_3 = 0$

$Max\ z = 11$ $\boxed{Min\ z = -11}$

Subject: operations Research
Subcode: 10ME74
SECTION: A & B

SCHEME & SOLUTION'S - IAT-1

1a. Let x_1, x_2 be the Number of Tons of Product X & Y to be produced resp.

$-1\times5=5$ MARKS

$$\text{Max } Z = 80x_1 + 120x_2$$

S.T

$x_1 + x_2 \le 9$ (Capacity Constraint)

$x_1 \ge 2$, $x_2 \ge 3$ (Supply Constraints)

$20x_1 + 50x_2 \le 360$ (M/chns Availability Constraint)

$x_1, x_2 \ge 0$

Coordinates of plot — $1\times3 = 3$ MARKS

$x_1 + x_2 = 9$ (9, 9)

$x_1 = 2$ (2, 0)

$x_2 = 3$ (0, 3)

$20x_1 + 50x_2 = 360$ (18, 7.2)

Points from Feasible Region	$\text{MAX } Z = 80x_1 + 120x_2$
A(2, 3)	520
B(2, 6.4)	928
C(3, 6)	960
D(6, 3)	840

3 Tons of X & 6 Tons of Y — To Achieve Maximum Profit of Rs 960/-

b. Assumptions a) Proportionality b) Additivity c) Divisibility d) Certainity e) finiteness f) optimality — $0.5\times4 = 2$ MARKS

3) Converting Minimisation Case into a Maximisation Case

$$Max\ Z = -4x_1 - x_2$$

Converting the inequalities into Equations by adding slack, Surplus & Artificial Variables

$3x_1 + x_2 + A_1 = 3$ — A_1, A_2 = Artificial Variables

$4x_1 + 3x_2 - S_1 + A_2 = 6$ — S_1 = Surplus Variable

$x_1 + 2x_2 + S_2 = 4$ — S_2 = Slack Variable.

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$ — 2 MARKS

4 iterations along with optimal Solution (4x2) — 8 MARKS

optimal Table:

	C_j	-4	-1	$-M$	0	$-M$	0		
C_B	Basis	x_1	x_2	A_1	S_1	A_2	S_2	RHS Constant	M.R
-4	x_1	1	0	2/5	0	0	−1/5	2/5	
-1	x_2	0	1	−1/5	0	0	3/5	9/5	
-1	S_1	0	0	1	1	−1	1	14/5	

$x_1 = 2/5$ $x_2 = 9/5$ $Max\ Z = -17/5$ $\boxed{Min\ Z = 17/5}$

4a) Explanation of slack, Surplus, Artificial Variables binding & Non-binding Constraints $1 \times 5 = 5$ MARKS

4b) Five Principles of Duality each 1 Mark $1 \times 5 = 5$ MARKS

5) Conversion of Primal to Dual

$Min\,W = 10y_1 + 6y_2 + 8y_3$

S.T

$y_1 + y_2 + 2y_3 \geq 2$

$5y_1 + 3y_2 + 2y_3 \geq 1$

$y_1, y_2, y_3 \geq 0$ of Dual variables — 1 MARK.

Solution using Simplex Method 4 iterations
1×4 – 4 MARKS
Solving Dual

optimal Table

C_B	Basis \ C_j	y_1 (−10)	y_2 (−6)	y_3 (−8)	S_1 (0)	A_1 (−M)	S_2 (0)	A_2 (−M)	RHS constant	M.R
0	S_2	−4	−2	0	−1	1	1	−1	1	
−8	y_3	$1/2$	$1/2$	1	$-1/2$	$1/2$	0	0	1	
	Z	6	2	0	4	M−4	0	M		

$x_1 =$ $\quad$ x_2

$y_1 = 0 \quad y_2 = 0 \quad y_3 = 1 \qquad Max\,W = -8 \qquad Min\,W = 8$

$x_1 = 4 \quad x_2 = 0 \qquad Max\,\cancel{Z} = -8 \qquad Min\,Z = 8$

Solution using Simplex Method Solving Primal

optimal Table:—

C_B	Basis \ C_j	x_1 (2)	x_2 (1)	S_1 (0)	S_2 (0)	S_3 (0)	RHS constant	M.R
0	S_1	0	4	1	0	$-1/2$	6	
0	S_2	0	2	0	1	$-1/2$	2	
2	x_1	1	1	0	0	$1/2$	4	
	Z	0	1	0	0	1		
				y_1	y_2	y_3		

2 iterations
2+2 = 4 MARKS

$x_1 = 4 \quad x_2 = 0$

$y_1 = 0 \qquad Max\,Z = 8$

$y_2 = 0 \qquad Min\,W = 8 \qquad$ FINAL Solution $\qquad$ 1 MARK

$y_3 = 1$

6a) Converting Minimization Case to Maximisation Case

$Max\,Z = -2x_1 - 2x_2 - 4x_3$

S.T

$-2x_1 - 3x_2 - 5x_3 \le -2$

$3x_1 + x_2 + 7x_3 \le 3$

$x_1 + 4x_2 + 6x_3 \le 5$

$x_1, x_2, x_3 \ge 0$ — 1 MARK

Converting the inequalities into Equations by adding slack Variables.

$-2x_1 - 3x_2 - 5x_3 + S_1 = -2$

$3x_1 + x_2 + 7x_3 + S_2 = 3$ $\quad S_1, S_2, S_3 =$ Slack Variables

$x_1 + 4x_2 + 6x_3 + S_3 = 5$ — 1 MARK

$x_1, x_2, x_3, S_1, S_2, S_3 \ge 0$

Solution using Dual Simplex Method 2 iterations

$2+2.5 = 5$ MARKS

Optimal Table:-

	c_j	-2	-2	-4	0	0	0	
C_b	Basis	x_1	x_2	x_3	S_1	S_2	S_3	RHS Constant
-2	x_2	$2/3$	1	$5/3$	$-1/3$	0	0	$2/3$
0	S_2	$7/3$	0	$16/3$	$1/3$	1	0	$7/3$
0	S_3	$-5/3$	0	$-2/3$	$4/3$	0	1	$7/3$

$x_1 = 0 \quad x_2 = 2/3 \quad x_3 = 0 \qquad Max\,Z = -4/3 \quad \boxed{Min\,Z = 4/3}$ — 1 MARK

6b) Explanation with Example infeasible Solution, unbounded Solution, Alternate optimal Solution, Degenerate Solutions $\quad 1 \times 4 = 4$ MARKS

7) Solution to Transportation problem using

North West Corner Rule : Rs 1800 — 2.5 MARKS

Row Minima Method : Rs 1750 — 2.5 MARKS

Column Minima Method : Rs 1725 — 2.5 MARKS

Matrix Method : Rs 1925 — 2.5 MARKS

8) Solution to Transportation Problem using Vogel's Approximation Method to

Determine IBFS : Rs 325 | Rs 330 | Rs 370 — 5 MARKS

optimality Test using MODI Method — 5 MARKS

optimal Solution : Rs 310

$u_1=0$	4 (1)	1 [45]	3 (1)	4 (1)	4 [15]
$u_2=-1$	2 [17]	3 (3)	2 (1)	2 [18]	3 (0)
$u_3=0$	3 [5]	5 (4)	2 [20]	4 (1)	4 [20]
	$v_1=3$	$v_2=1$	$v_3=2$	$v_4=3$	$v_5=4$

Min Transportation Cost = Rs 310/-