

INTERNAL ASSESSMENT TEST I

SEPT-2017

(Solution Key)

1 a) Describe circular polariscope identify all its components and derive expression for the intensity of the light wave in a dark field arrangement [10M]

- Draw the diagram (3M)
- Derive the final expression (7M)

2 a) Write short notes on Isoclinics & Isochromatics. [5M]

- Draw diagram for each (2M)
- Write detailed explanation (3M)

b) A fringe order of 2.5 was observed at a point in a stressed model with light having $\lambda = 589\text{nm}$. What fringe order is observed at the point in consideration when light with $\lambda = 548\text{nm}$ is used. (5M)

- Derive equation for fringe order (2M)
- Calculation - fringe order = 2.68 (3M)

3 a) What are the properties of an ideal photoelastic material? Discuss applications and name few photoelastic materials. (10M)

- Write properties (4 marks)
- Application (3M)
- Example (2M)

4(a) Define stress optics law. And derive it in two dimensional photoelasticity. (5M)

- State the stress optics law (2M)
- Write all equations (2M)
- Rewrite equation for 2 dimensional photoelasticity (1M)

b) Write a short note on waveplates. (5M)

- Explain wave theory (2M)
- Write about all different type of wave plate (3M)

5) a) Explain polarization and the working principle of a polariscope. (5M)

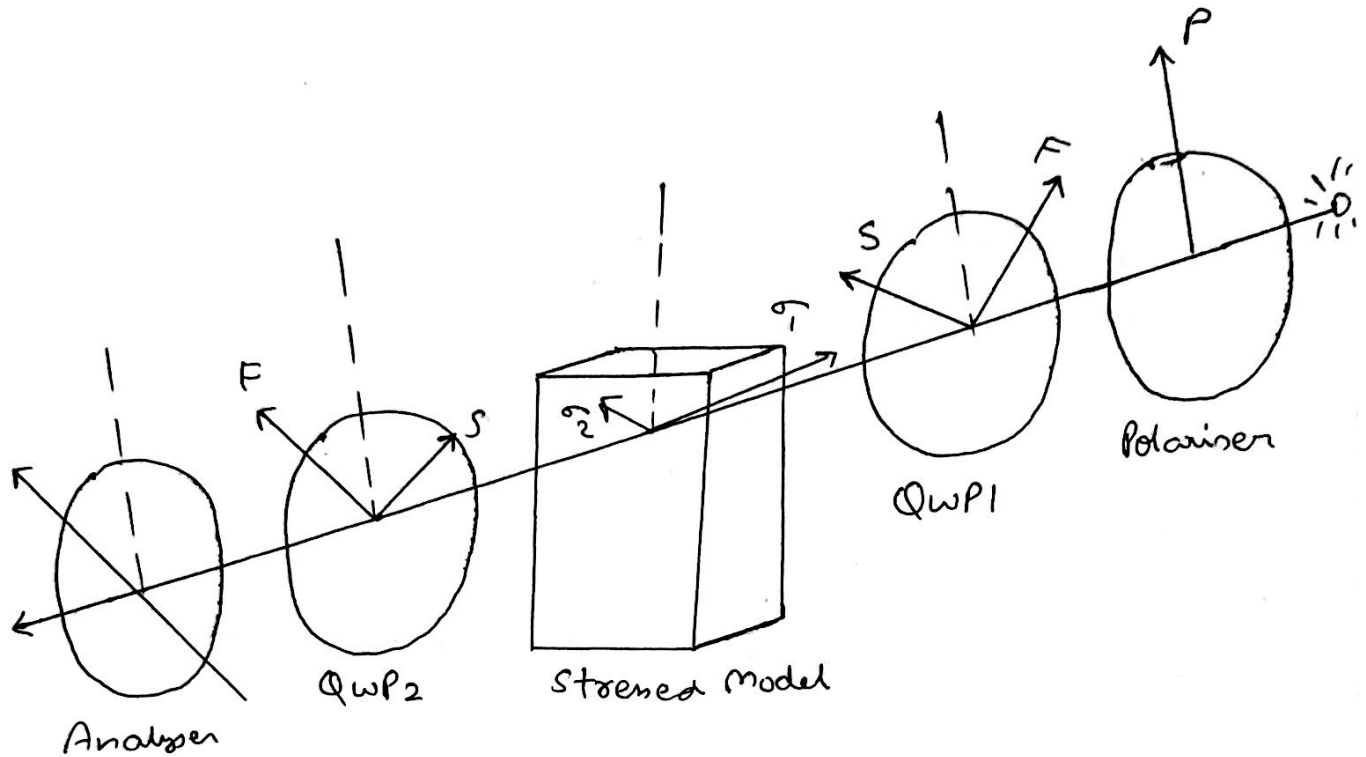
- Write the detailed explanation for polarization process (3M)
- Principle of polariscope (2M)

b) Explain the importance of passage of light through crystalline medium. (5M)

- Diagram for light through crystalline medium (2M)
- Write the complete process of light through medium, (3M)

IAT-1 SOLUTION

1a)



Let plane polarised light coming out of the polarizer be,

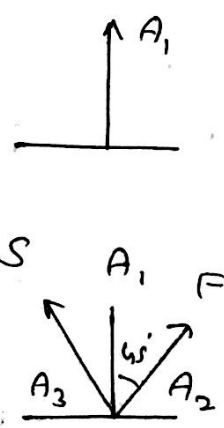
$$A_1 = a \cos \omega t$$

QWP1: At entry $A_2 = A_1 \cos 45^\circ = \frac{a}{\sqrt{2}} \cos \omega t$

$$A_3 = A_1 \sin 45^\circ = \frac{a}{\sqrt{2}} \cos \omega t$$

At exit $A_4 = A_2 = \frac{a}{\sqrt{2}} \cos \omega t$

$$A_5 = \frac{a}{\sqrt{2}} \cos(\omega t + \frac{\pi}{2}) = -\frac{a}{2} \sin \omega t$$

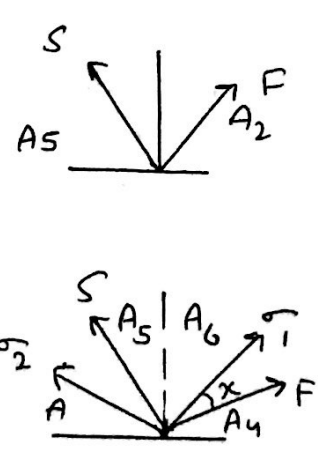


Stressed model

At entry,

$$A_6 = A_4 \cos \alpha + A_5 \sin \alpha$$

$$= \frac{a}{\sqrt{2}} \cos \omega t \cdot \cos \alpha - \frac{a}{\sqrt{2}} \sin \omega t \cdot \sin \alpha$$



$$= \frac{a}{\sqrt{2}} \cos(\omega t + \alpha)$$

$$A_7 = A_5 \cos \alpha - A_4 \sin \alpha = -\frac{a}{\sqrt{2}} \sin \omega t \cdot \cos \alpha - \frac{a}{\sqrt{2}} \cos \omega t \cdot \sin \alpha$$

$$= -\frac{a}{\sqrt{2}} \sin(\omega t + \alpha)$$

At exit.

$$A_8 = A_6 = \frac{a}{\sqrt{2}} \cos(\omega t + \alpha)$$

$$A_9 = \frac{a}{\sqrt{2}} \sin(\omega t + \alpha + \delta)$$

QWP₂

At entry

$$A_{10} = A_8 \cos \alpha - A_9 \sin \alpha$$

$$= \frac{a}{\sqrt{2}} \cos(\omega t + \alpha) + \frac{a}{\sqrt{2}} \sin(\omega t + \alpha + \delta) \sin \alpha$$

$$A_{11} = A_8 \sin \alpha + A_9 \cos \alpha$$

$$= \frac{a}{\sqrt{2}} \cos(\omega t + \alpha)$$

At exit.

$$A_{12} = \frac{a}{\sqrt{2}} \cos(\omega t + \alpha + \frac{\pi}{2}) \cos \alpha + \frac{a}{\sqrt{2}} \sin(\omega t + \alpha + \delta + \frac{\pi}{2}) \sin \alpha$$

$$= -\frac{a}{\sqrt{2}} \sin(\omega t + \alpha) \cos \alpha + \frac{a}{\sqrt{2}} \cos(\omega t + \alpha + \delta) \sin \alpha$$

$$A_{13} = A_{11} = \frac{a}{\sqrt{2}} \cos(\omega t + \alpha) \sin \alpha - \frac{a}{\sqrt{2}} \sin(\omega t + \alpha + \delta) \cos \alpha$$

Analyzer

At exit

$$A_{14} = A_{13} \sin 45 - A_{12} \sin 45 \quad \sin 45 = \frac{1}{\sqrt{2}}$$

$$= \frac{a}{2} [-\sin(\omega t + \alpha + \delta) \cos \alpha + \cos(\omega t + \alpha) \sin \alpha + \sin(\omega t + \alpha) \cos \alpha]$$

$$- \cos(\omega t + \alpha + \delta) \sin \alpha$$

$$= \frac{a}{2} [\sin(\omega t + \alpha + \delta + \alpha) - \sin(\omega t + \alpha + \alpha)]$$

$$= \frac{a}{2} [\sin(\omega t + 2\alpha + \delta) - \sin(\omega t + 2\alpha)]$$

$$= \frac{a}{2} [\sin(\omega t + 2\alpha) \cdot \cos \delta + \cos(\omega t + 2\alpha) \sin \delta - \sin(\omega t + 2\alpha)]$$

$$= \frac{a}{2} [\sin(\omega t + 2\alpha)(\cos \delta - 1) + \cos(\omega t + 2\alpha) \sin \delta]$$

$$= -\frac{a}{2} [\sin(\omega t + 2\alpha) 2 \sin^2 \frac{\delta}{2} - \cos(\omega t + 2\alpha) 2 \sin \frac{\delta}{2} \cos \frac{\delta}{2}]$$

$$= -\frac{a}{2} 2 \sin \frac{\delta}{2} [\sin(\omega t + 2\alpha) \sin \frac{\delta}{2} - \cos(\omega t + 2\alpha) \cos \frac{\delta}{2}]$$

$$= -a \sin \frac{\delta}{2} [-\cos(\omega t + 2\alpha + \frac{\delta}{2})]$$

$$= +a \sin \frac{\delta}{2} \cos(\omega t + 2\alpha + \frac{\delta}{2})$$

intensity $I = A \sin^2 \frac{\delta}{2}$

Extinction ($I=0$) occurs when $\frac{\delta}{2} = n\pi$, $n=0, 1, 2, 3, \dots$

or

when Fringe order $N = \frac{\delta}{2\pi} = n$, $n=0, 1, 2, 3, \dots$

which implies that the order of the 1st fringe observed in a dark field polariscope setup is zero for $n=0$.

2a)

Two different types of fringes can be observed in photoelasticity: isochromatic and isoclinic fringes. Isochromatic fringes are lines of constant principle stress difference, $(\sigma_1 - \sigma_2)$. If the source light is monochromatic these appear as dark and light fringes, whereas with white light illumination coloured fringes are observed. The difference in principle stress is related to the birefringence and hence the fringe colour through the stress-optic law.

Isoclinic fringes occur whenever either principle stress direction coincides with the axis of polarisation or the polariser, isoclinic fringes therefore provide info about the directions of the principle stresses in the model. When combined with the values of $(\sigma_1 - \sigma_2)$ from the photoelectric stress pattern, isoclinic fringes provide the necessary information for the complete solution of a two-dimensional stress problem.

A standard plane polariscope shows both isochromatic and isoclinic fringes and this makes quantitative stress analysis difficult.

3

2 b) $N_1 = 2.5$ we know that, the relative phase diff

$$\begin{array}{l|l} \lambda_1 = 589 \text{ nm} & \delta = \frac{2\pi hc}{\lambda} (\sigma_1 - \sigma_2) \\ \lambda_2 = 548 \text{ nm} & \end{array}$$

Fringe order per no or wavelengths or relative path

difference, $N = \frac{\delta}{2\pi} = \frac{hc}{\lambda} (\sigma_1 - \sigma_2)$ or $hc (\sigma_1 - \sigma_2) = N\lambda$

For a particular model and loading conditions c, h, σ_1 & σ_2 are constant.

$$\therefore N\lambda = \text{constant}$$

or

$$N_1 \lambda_1 = N_2 \lambda_2$$

$$N_2 = \frac{N_1 \lambda_1}{\lambda_2} = \frac{2.5 \times 589}{548} = 2.687.$$

3a) Properties of an ideal photo elastic material:

Optical:

- Model material must be transparent
- It must have high sensitivity in order that a large number of fringes are observed.
- It must exhibit linear characteristics stress-strain, stress-fringe order.

Structural

- material must be isotropic and homogeneous

Mechanical

- Material to distort lens, it should have high modulus of elasticity and high ultimate strength
- Poisson's ratio of the specimen and prototype must be as close as possible
- must be better creep resistant
- Material constant must not vary with temperature
- Exothermic reaction should be low during mixing and setting process.

Production

- It should have good castability and machinability

Cost

- Cost of the material should be low.

few important photoelastic materials are.

1) Homalite-100 or Castolite:

It is a polyester resin cast between two plates of glass. less creep.

2) polycarbonate:

A thermoplastic known by the trade name Iescan which has high sensitivity, exhibits very little creep and relatively free from edge effects.

3) Polyurethane Rubber:

Polyurethane Rubber has very low modulus of elasticity, very high sensitivity, negligible time or edge effects and can easily be machined.

4a) Define stress optic law

Stress optic law states that, the changes in the indices of refraction of a material exhibiting double refraction or birefringence can be related to the state of stress in material as.

$$\left. \begin{aligned} n_1 - n_0 &= C_1 \sigma_1 - C_2 \sigma_2 \\ n_2 - n_0 &= C_1 \sigma_2 - C_2 \sigma_1 \end{aligned} \right] \text{--- (1)}$$

if a plane polarised light is incident normally at any point P of the model, then the incident light vector gets resolved along the directions of principle stresses σ_1 & σ_2 travel through the model with different velocities and when they emerge, there will be a relative phase difference given by

$$\delta = \frac{2\pi h}{\lambda} (n_1 - n_2)$$

from (1)

$$n_1 - n_2 = (C_1 + C_2) (\sigma_1 - \sigma_2)$$

$$\therefore \delta = \frac{2\pi h}{\lambda} (C_1 + C_2) (\sigma_1 - \sigma_2)$$

$$\text{or } \delta = \frac{2\pi h}{\lambda} C (\sigma_1 - \sigma_2)$$

Fringe order or the number of wavelengths

$$N = \frac{\delta}{2\pi} = \frac{h}{\lambda} C (\sigma_1 - \sigma_2)$$

$$\Rightarrow (\sigma_1 - \sigma_2) = \frac{\lambda}{Ch} \cdot N$$

$$\text{or } \sigma_1 - \sigma_2 = \frac{F}{h} N$$

$$f = \frac{\lambda}{Ch} \text{ model fringe constant}$$

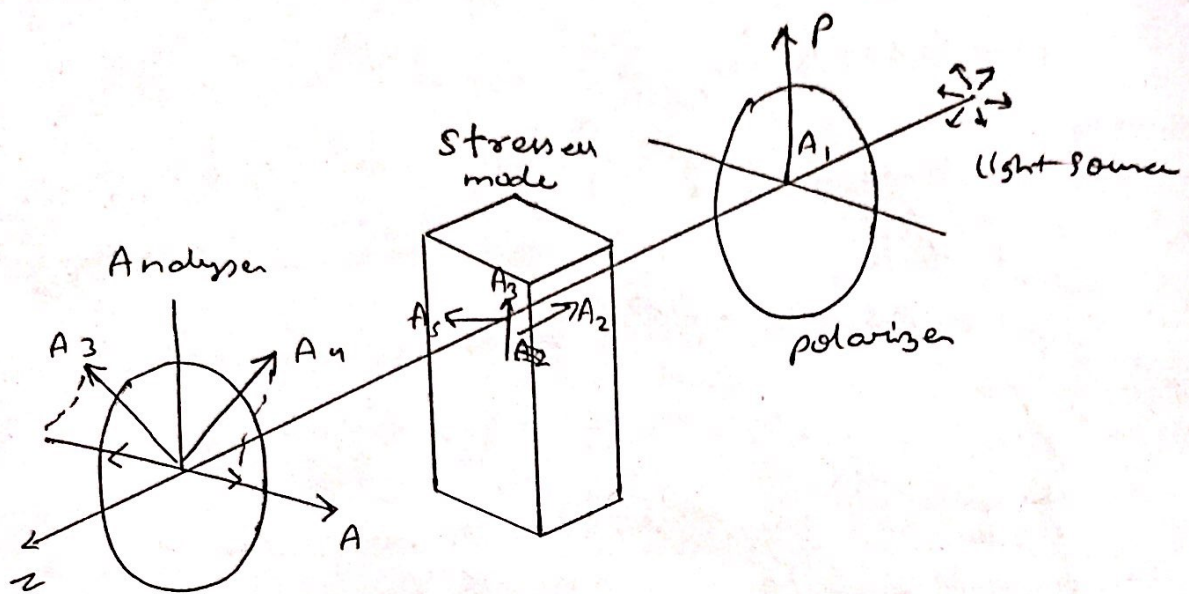
$$F = \frac{\lambda}{C} \text{ is call fringe material constant.}$$

4b) A wave plate or retarder is an optical device that alters the polarization state of a light wave travelling through it. Two common types of waveplates are the half-wave plate, which shifts the polarization direction of linearly polarized light, and the quarter-wave plate, which converts linearly polarized light into circularly polarized light and vice versa.

A quarter-wave plate can be used to produce elliptical polarization.

By appropriate choice of the relationship between these parameters, it is possible to introduce a controlled phase shift between the two polarization components of a light wave, thereby altering its polarization.

5 a)



$$A_1 = a \cos \omega t$$

light along principle streu axen.

$$A_2 = A_1 \cos \alpha = a \cos \omega t \cdot \cos \alpha$$

$$A_3 = A_1 \sin \alpha = a \cos \omega t \cdot \sin \alpha$$

$$A_4 = a (\cos \alpha \cos(\omega t + \delta))$$

$$A_5 = A_3 = a \sin \alpha \cos \omega t.$$

$$A_6 = A_4 \sin \alpha - A_5 \cos \alpha$$

$$= a \cos \alpha \cdot \sin \alpha \cdot \cos(\omega t + \delta) - a \cos \alpha \sin \alpha \cos \omega t$$

$$= a \frac{\sin 2\alpha}{2} [\cos(\omega t + \delta) \cos \omega t]$$

$$= \frac{a}{2} \sin 2\alpha (\cos \omega t \cdot \cos \delta - \sin \omega t \cdot \sin \delta - \cos \omega t)$$

$$= \frac{a}{2} \sin 2\alpha (\cos \omega t (\cos \delta - 1) - \sin \omega t \cdot \sin \delta)$$

$$= \frac{a}{2} \sin 2\alpha [\cos \omega t (-2 \sin^2(\delta/2)) - 2 \sin \omega t \cdot \sin(\delta/2) \cos(\delta/2)]$$

$$= -a \sin 2\alpha \cdot \sin(\delta/2) [\cos \omega t \cdot \sin(\delta/2) + \sin \omega t \cdot \cos(\delta/2)]$$

$$= -a \sin 2\alpha \cdot \sin(\delta/2) \sin(\omega t + \delta/2)$$

$$x \sin(\omega t + \delta/2)$$

$$= b \sin(\omega t + \delta/2)$$

where

$$b = a \sin 2 \alpha \sin\left(\frac{\delta}{2}\right)$$

$$1 \propto b^2$$

$$1 = A \sin^2 2 \alpha \sin^2\left(\frac{\delta}{2}\right)$$

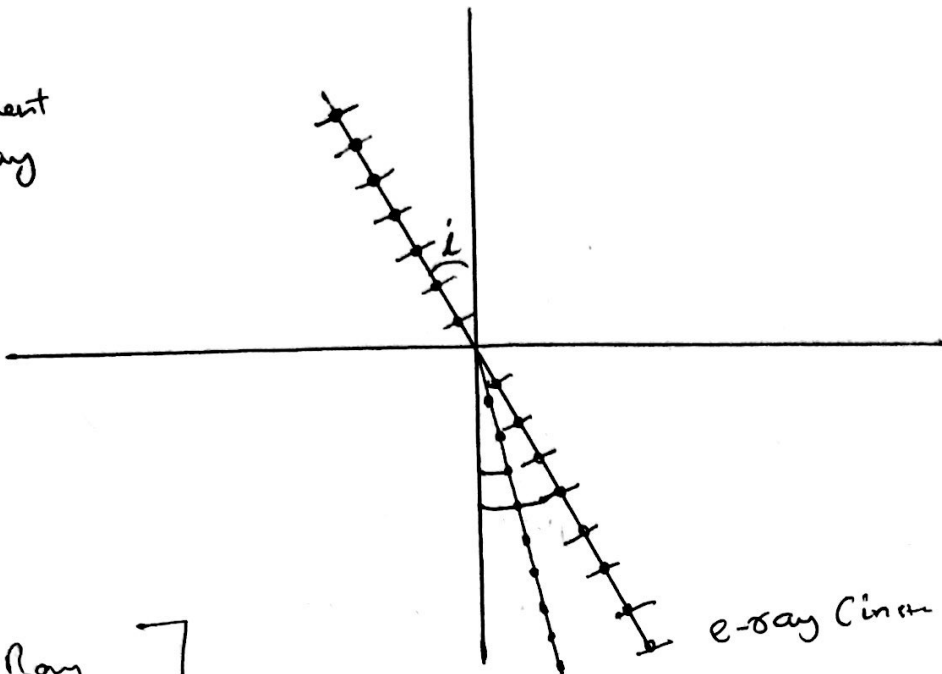
5b) If a plane polarised beam of light is incident normally on the surface of a plane parallel uniaxial crystal plate. There will be two emergent rays one with the plane of vibration perpendicular to principal section and the other (E) with in the principle section.

However the plane of vibration of the incident light is ~~on~~ perpendicular to principle section there will be one plane polarized emergent beam.

These two rays have different velocities and are called privileged directions, slow and fast axis.

- ① Crystalline are optically anisotropic.
- ② A single incident ray, will give rise to two refracted rays, ordinary 'O' and known as double refraction.
- ③ Extraordinary rays manage to violate Snell's law under suitable circumstances.
- ④ An Isotropic medium can transmit common light while the light travelling through a crystal is always polarized
- ⑤ The ordinary and extraordinary rays are plane polarized and their planes of polarization are \perp to each other.

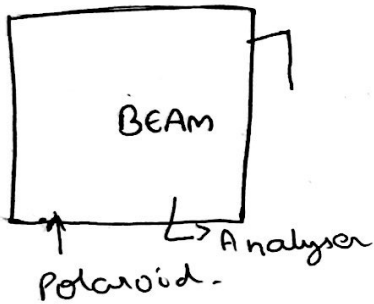
Incident Ray



[1-Ray
diff. vel
diff. dirⁿ]

e-ray (linear vertical)

o-ray
(linear horizontal)



By placing an analyser appropriately
one of the images is eliminated
This shows that that extinguished
image is plane polarised
perpendicular to analyser.

⇒ Refractive Indices of various crystals

Crystal	n_o	n_e	$n_e - n_o$
Ice	1.309	1.313	0.004
Calcite	1.658	1.486	-0.172