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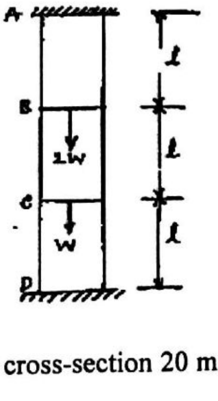
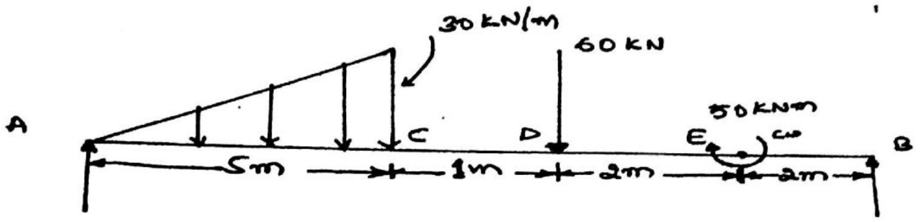
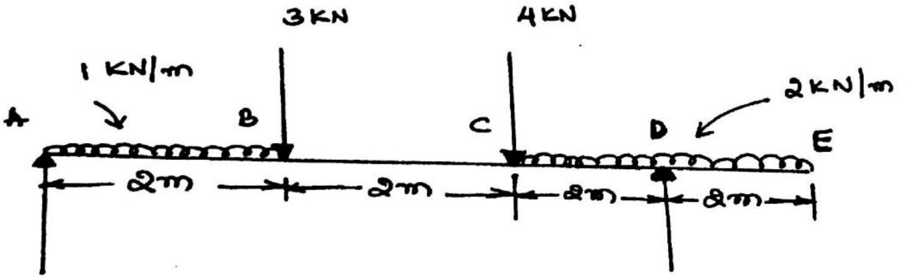
Internal Assesment Test – 1

Sub: Mechanics of Materials				Code: 15ME34
Date: 20/09/2017	Duration: 90 mins	Max Marks: 50	Sem: 3	Branch (sections): ME (A,B)

Answer any four questions from Part A and ONE question from Part B. Good luck!

	PART A	Marks	OBE CO	OBE RBT
1	Two vertical rods one of steel and the other of copper are each rigidly fixed at the top and 500mm apart. Diameters and lengths of each rod are 20mm and 4m respectively. A cross bar fixed to the rods at the lower ends carries a load of 5kN, such that the cross bar remains horizontal even after loading. Find the stress in each rod and the position of the load on the bar. Take Es = $2x10^5$ N/mm^2 and Ec = $1x10^5$ N/mm^2.	[10]	CO2	L3
2	A steel tube of 25mm external diameter and 18mm internal diameter encloses a copper rod of 15mm diameter. The ends are rigidly fastened to each other. Calculate the stress in the rod and the tube when the temperature is raised from 15° to 200°C. Take E_s = 200 GPa ; E_c = 100 GPa; αs= $11x10^{-6}$ /°C ; αc= $18x10^{-6}$ /°C.	[10]	CO2	L3
3 (a)	Sketch and explain stress-strain diagram for steel indicating all salient points and zones on it.	[05]	CO1	L2
(b)	Derive an expression for volumetric strain of a rectangular block subjected to axial load.	[05]	CO1	L2
4	Rails are laid such that there is no stress in them at 24°C. If the rails are 32 m long, Determine: i. The stress in the rails at 80°C, when there is no allowance for expansion ii. The stress in the rails at 80°C. When there is an expansion allowance of 8 mm per rail. iii. The expansion allowance for no stress in the rails at 80°C. Coefficient of linear expansion,α = 11 x 10^{-6}/°C and Young's modulus E = 205 GPa.	[10]	CO2	L2
5	A vertical circular steel bar of length 3l fixed at both of its ends is loaded at intermediate sections by forces W and 2W as shown in Fig. below. Determine the end reactions if W= 2.25 kN.  	[10]	CO2	L3
6	A 500 mm long bar has rectangular cross-section 20 mm * 40 mm. This bar is subjected to i. 40 kN tensile force on 20mm × 40 mm faces, ii. 200 kN compressive force on 20 mm × 500 mm faces, and iii. 300 kN tensile force on 40 mm × 500 mm faces. Find the change in volume if E= 2 × 10^5 N/mm^2 and μ= 0.3.	[10]	CO2	L3

PART B

7	Draw shear force diagram & bending moment diagram for the simply supported beam shown in figure.  	[10]	CO4	L3
8	Draw shear force diagram & bending moment diagram for the overhanging beam shown in figure.  	[10]	CO4	L3

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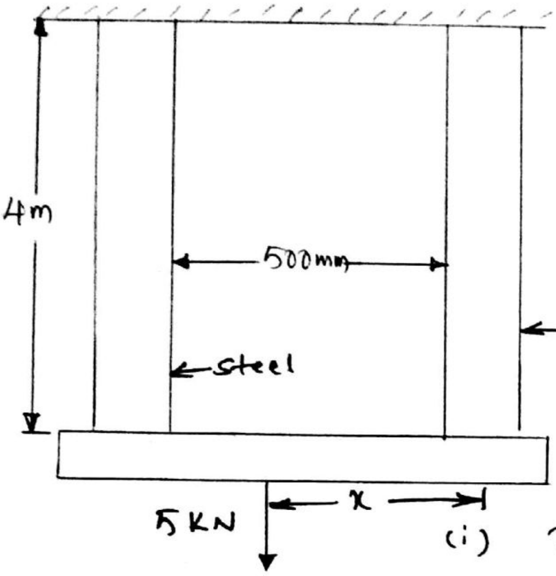
Department of Mechanical Engineering.

INTERNAL ASSESMENT - TEST 1 Sem: 3rd (A & B)

Date: 20/09/2017 Subject: Mechanics of Materials (Solutions) Subject code: 15ME34

Max Marks: 50

1\.



Data: $E_s = 2\times10^5$ N/mm^2, $E_c = 1\times10^5$ N/mm

Diameters of Steel & Copper : 2cm

Length of each rod : $\underline{4m} = 4000$ mm

Area of steel rod = Area of Copper rod

$$= \frac{\pi}{4} \times 20^2$$

$$A_s = A_c = 100\pi \text{ mm}^2$$

$$\sigma_s = ?\ ,\ \sigma_s = ?$$

(i) To determine σ_s and σ_c.

Since the cross bar remains horizontal, the extensions of the steel and copper rods are equal. Also the rods have the same original length, hence the strains of these rods are equal.

$\therefore$ Strain in steel = Strain in Copper

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\therefore \sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{2\times10^5}{1\times10^5} \times \sigma_c = \underline{2\times\sigma_c} \quad -(i)$$

We know that, Total load = Load on steel rod + Load on Copper ro

$$5000\text{N} = P_s + P_c$$

$$5000 = \sigma_s \cdot A_s + \sigma_c \cdot A_c$$

$$= 2\sigma_c \times 100\pi + \sigma_c \times 100\pi$$

$$\therefore \sigma_c = \frac{5000}{300\pi} = \underline{5.305 \text{ N/mm}^2} \quad \text{[Ans]}$$

Substituting this value of σ_c in eqn (i)

$$\sigma_s = 2\times\sigma_c = \underline{10.61 \text{ N/mm}^2} \quad \text{[Ans]}$$

(ii) Position of the load of 5000 N on cross bar.

Let, x = distance of 5000 N load from the copper rod.

Load carried by each rod:

w.k.T Load = stress × Area

Load carried by steel	Load carried by copper
$P_s = \sigma_s \times A_s$	$P_c = \sigma_c \times A_c$
	or
$= 10.61 \times 100\pi$	$P = P_s + P_c$
$= \underline{3333 \text{ N}}$	or $P_c = P - P_s$
	$= 5000 - 3333$
	$P_c = \underline{1667 \text{ N}}$

Now taking moments about the copper rod and equating the same, we get:

$$5000 \cdot x = P_s \times 50$$

$$\therefore x = \frac{3333 \times 50}{5000}$$

$$\therefore x = \underline{33.33 \text{ Cm}} \quad [\text{Ans}]$$

2. Data: $E_s = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2 = 200 \times 10^3 \text{ N/mm}^2$

$E_c = 100 \text{ GPa} = 100 \times 10^9 \text{ N/m}^2 = 100 \times 10^3 \text{ N/mm}^2$

$\alpha_s = 11 \times 10^{-6}/°c$; $\alpha_c = 18 \times 10^{-6}/°c$; $T = T_2 - T_1$

$= 200 - 15$

$= 185 °c$

Area of copper rod, $A_c = \frac{\pi}{4} \times 15^2 = \underline{56.25\pi} \text{ mm}^2$

Area of steel tube, $A_s = \frac{\pi}{4}(25^2 - 18^2) = \underline{75.25\pi} \text{ mm}^2$

As the free expansion of copper is more than free expansion of steel, Compressive stress will be induced in copper and tensile stress will be induced in steel to keep them in the same position.

For equilibrium of the system

Compressive load on copper = Tensile load on steel

$$F_c = F_s$$

$$\sigma_c A_c = \sigma_s \cdot A_s$$

$$\sigma_c \times 56.25\pi = \sigma_s \times 75.25\pi$$

$$\therefore \sigma_c = 1.3378\,\sigma_s \quad -(i)$$

Actual expansion of steel = Free expansion of steel + Expansion due to tensile - stress

$$= \alpha_s T l_s + \frac{P_s l_s}{A_s E_s}$$

Actual expansion of copper = Free expansion of copper - contraction due to compressive stress

$$= \alpha_c T l_c - \frac{P_c l_c}{A_c E_c}$$

Since, Actual expansion of steel tube = Actual expansion of copper rod

$$\alpha_s T \cancel{l_s} + \frac{P_s \cancel{l_s}}{A_s E_s} = \alpha_c T \cancel{l_c} - \frac{P_c \cancel{l_c}}{A_c E_c}$$

$\therefore (l_c = l_s = l)$

~~$\alpha_s \times 185 \times l$ +~~

$$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c}$$

$\left(\because \frac{P_s}{A_s} = \sigma_s\right.$

$\left(\because \frac{P_c}{A_c} = \sigma_c\right)$

$$\frac{\sigma_s}{E_s} + \frac{\sigma_c}{E_s} = (\alpha_c - \alpha_s)\,T$$

$$\frac{\sigma_s}{200\times10^3} + \frac{1.3378\,\sigma_s}{100\times10^3} = (18\times10^{-6} - 11\times10^{-6})\,185$$

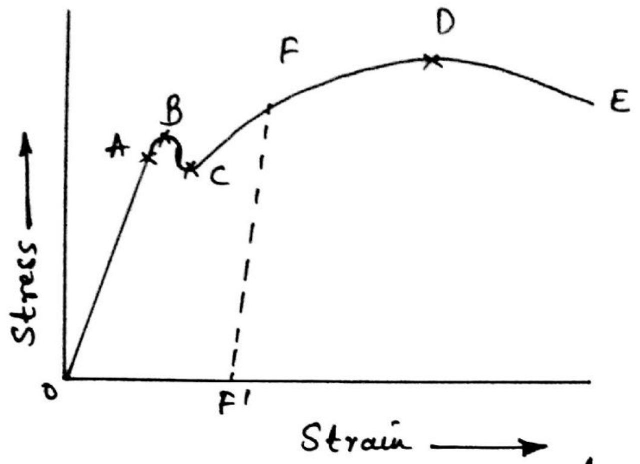
$\therefore$ stress induced in steel tube, $\sigma_s = \underline{70.465\ N/mm^2}$ (Tensile) [Ans]

Stress induced in copper rod, $\sigma_c = 1.3378 \times 70.465$

$= \underline{94.27\ N/mm^2}$ (Compressive) [Ans]

3
(a)

Stress- strain diagram (Steel)



(a) Limit of proportionality (A): It is the limiting value of stress up to which stress is proportional to strain.

(b) Elastic limit: This is the limiting value of stress up to which if the material is stressed and then released (unloaded) strain disappears completely and the original length is regained. This point is slightly beyond the limit of proportionality.

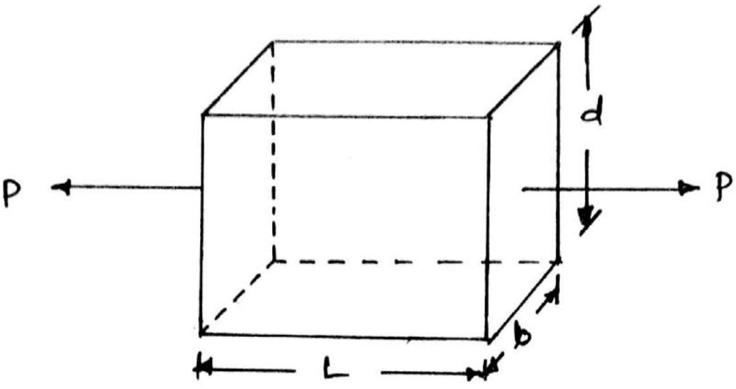
(c) Upper yield point (B): This is the stress at which, load starts reducing and the extension increases. This phenomenon is called yielding of material.

(d) Lower yield point (C): At this stage the stress remains same but strain increases for some time.

(e) Ultimate stress (D): This is the maximum stress the material can resist. At this stage cross sectional area at a particular section starts reducing very fast. This is called neck formation. After this stage load resisted and hence the stress developed starts reducing.

(f) Breaking point (E): The stress at which finally the specimen fails is called breaking point.

3
(b).



Let,
δl = Change in length
δb = Change in width &
δd = Change in depth

Final length of the bar = $L + \delta l$

Final width of the bar = $b - \delta b$ (-ve sign due to decrease in width)

Final depth of the bar = $d - \delta d$ (-ve sign due to decrease in depth)

Now, original volume of bar, $V = L \cdot b \cdot d$

Final volume = $(L+\delta l)\ (b-\delta b)\ (d-\delta d)$

$= Lbd + bd\delta l - Lb\delta d - Ld\delta b$ (ignoring products of small quantities

∴ Change in volume,

δv = Final volume − Original volume

$= (Lbd + bd\delta l - Lb\delta d - Ld\delta b) - Lbd$

$= bd\delta l - Lb\delta d - Ld\delta b$

w.k.t volumetric strain $e_v = \frac{\delta v}{V} = \frac{bd\delta l - Lb\delta d - Ld\delta b}{Lbd}$

$= \frac{\delta L}{L} - \frac{\delta d}{d} - \frac{\delta b}{b}$ —(i)

But, $\frac{\delta l}{L}$ = Longitudinal strain and $\frac{\delta d}{d}$ or $\frac{\delta b}{b}$ = Lateral strain

Substituting these values in the above eqn, we get

e_v = Longitudinal strain − 2 × Lateral strain — (ii)

We know that, $\dfrac{\text{Lateral Strain}}{\text{Longitudinal strain}} = \mu$ (Poisson's ratio)

$\therefore$ Lateral strain $= \mu \times$ longitudinal strain

Substituting the value of lateral strain in eqn (ii)

$e_v =$ longitudinal strain $- 2 \times \mu$ longitudinal strain

$e_v =$ longitudinal strain $(1 - 2\mu)$

$$\therefore e_v = \frac{\delta l}{l}(1 - 2\mu)$$

or

Volumetric strain, $e_v = \dfrac{\delta l}{l}\left(1 - \dfrac{2}{m}\right)$ $\quad \left(\because \dfrac{1}{m} = \mu\right)$

4. **Data:**

$L = 32\,m$, $T_1 = 24°c$, $T_2 = 80°c$, $\alpha = 11 \times 10^{-6}/°c$, $E = 205GPa$.

$T = T_2 - T_1 = 80 - 24 = 56°c$

(i) Stress in rails, when there is no allowance for expansion.

$$\sigma_t = \alpha T E = (11 \times 10^{-6}) \times 56 \times 205 \times 10^3$$

$$\therefore \sigma_t = 126.28\ N/mm^2 \text{ [Ans]}$$

(ii) Stress when there is an expansion allowance of 8mm

$$\sigma_t = \frac{(\alpha T L - \delta)}{L} \times E$$

$$= \frac{[(11 \times 10^{-6}) \times 56 \times 205 \times 10^3] - 8}{32 \times 10^3} \times 205 \times 10^3$$

$$= 75.03\ N/mm^2 \quad \text{[Ans]}$$

(iii) Expansion allowance for no stress in the rails at 80°c.

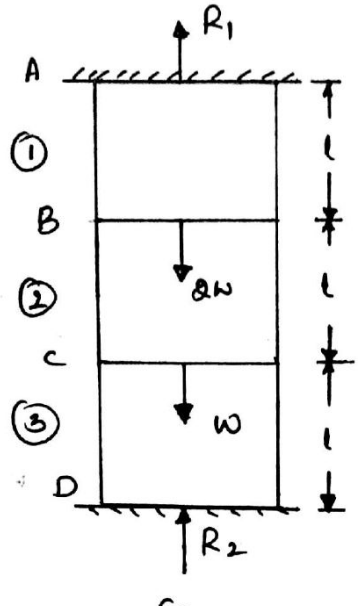
Free expansion of the rail $= \alpha T l = 11 \times 10^{-6} \times 56 \times \cancel{205 \times 10^3}\ 32 \times 10^3$

$= 19.712\ mm$ [Ans]

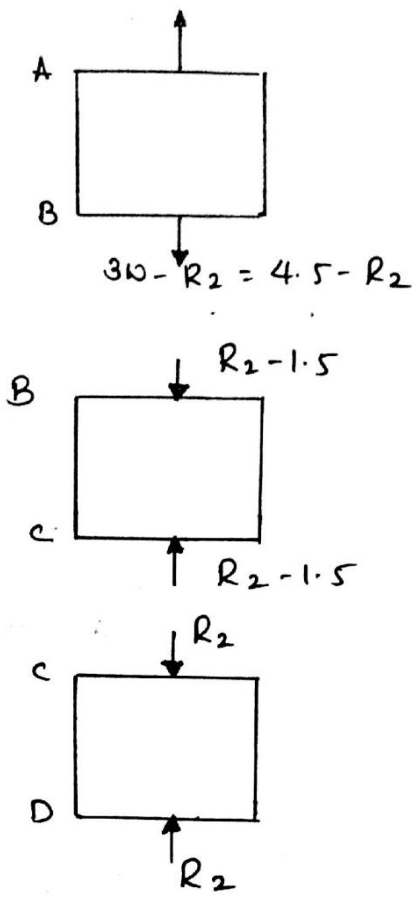
5\. Data: $w = 2.25$ kN

Free body diagrams

$R_1 = 4.5 - R_2$



(a)



(b)

For equilibrium

$R_1 + R_2 = 2w + w = 3w = 3 \times 1.5$

$\therefore R_1 = 4.5 - R_2$

Since both ends are fixed, total change in length, $\delta = 0$

Change in length of section AB, $\delta_{AB} = \frac{Pl}{AE} = \frac{(4.5 - R_2)\,l}{A_{AB} \cdot E}$ (Elongation)

Change in length of section. BC, $\delta_{BC} = \frac{Pl}{AE} = \frac{(R_2 - 1.5)\,l}{A_{BC} \cdot E}$ (contraction)

Change in length of section CD, $\delta_{CD} = \frac{Pl}{AE} = \frac{R_2\, l}{A_{CD} \cdot E}$ (contraction)

As, $\delta = 0$, $\delta_{AB} = \delta_{BC} + \delta_{CD}$

$$\frac{(4.5 - R_2)\,l}{A_{AB} \cdot E} = \frac{(R_2 - 1.5)\,l}{A_{BC} \cdot E} + \frac{R_2\, l}{A_{CD} \cdot E} \qquad (\because A_{AB} = A_{BC} = A_{CD})$$

$4.5 - R_2 = R_2 - 1.5 + R_2$

$4.5 + 1.5 = 3R_2$

$\therefore \underline{R_2 = 2\text{ kN}}$ [Ans] and $R_1 = 4.5 - R_2 = 4.5 - 2$

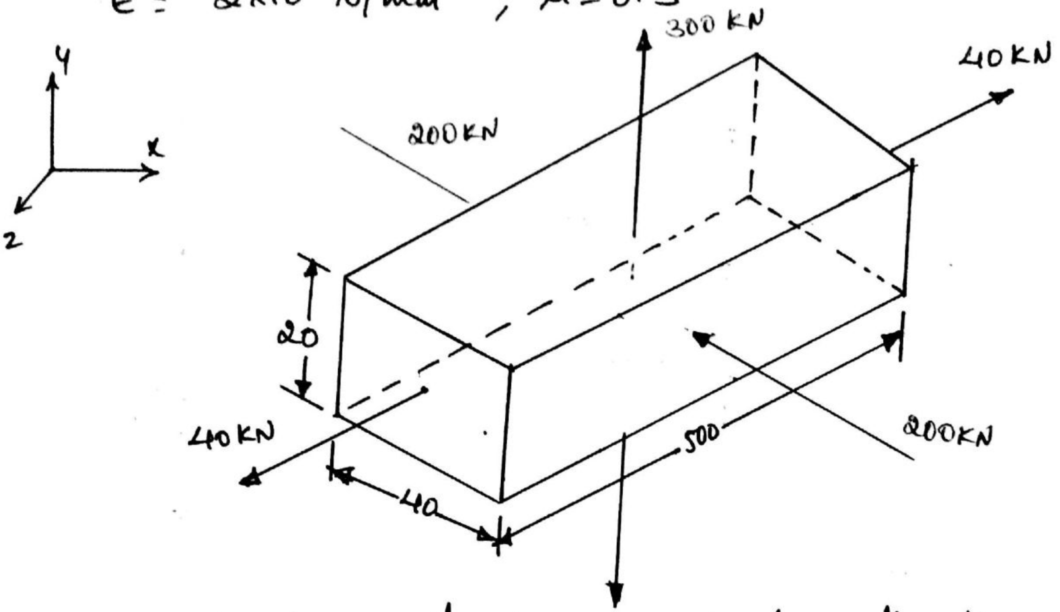
$\therefore R_1 = \underline{2.5\text{ kN}}$ [Ans]

6. Data : 40 KN - 200mm x 40mm face (Tensile)

200 KN - 20mm x 500mm face (Compressire)

300 KN - 40 mm x 500mm face (~~Compressire~~ Tensile)

$E = 2\times10^5\ N/mm^2$, $\mu = 0.3$



Let x, y, z be the mutually perpendicular directions.

Now, $\sigma_x = \frac{P\cancel{A}}{A} = \frac{40\times10^3}{20\times40} = 50\ N/mm^2$ (Tensile)

$\sigma_y = \frac{P}{A} = \frac{200\times10^3}{20\times500} = 20\ N/mm^2$ (Compressive)

$\sigma_z = \frac{P}{A} = \frac{300\times10^3}{40\times500} = 15\ N/mm^2$ (Tensile)

~~Net strain along 'x' direction~~

$$\cancel{e_x = \frac{\sigma_x}{E} + \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}}$$

(or)

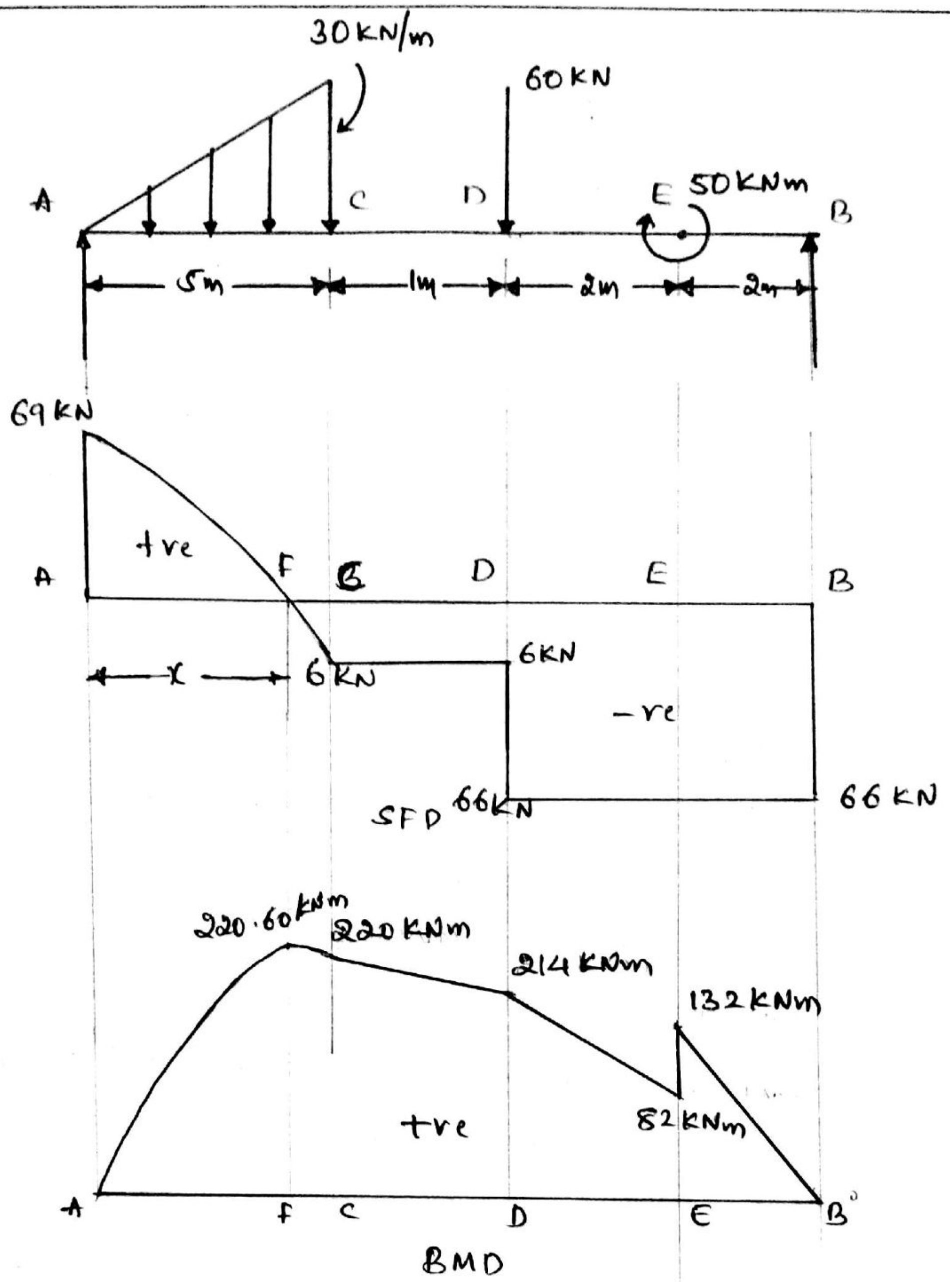
Volumetric strain $e_v = \frac{\delta v}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1-2\mu)$

$$\frac{\delta v}{V} = \frac{1}{2\times10^5}(50-20+15)(1-2\times0.3)$$

$$\delta v = \frac{(20\times40\times500)(50-20+15)(1-0.6)}{2\times10^5}$$

$\therefore\ \delta v = \underline{36\ mm^3}$ [Ans]

7.



To determine reactions at supports (R_A & R_B):

Taking moments about A, we get:

$$R_B \times 10 - 60 \times 6 - (1/2 \times 30 \times 5)(2/3 \times 5) - 50 = 0$$

$$R_B \times 10 = 360 + 50 + 250$$

$$R_B = \frac{660}{10} = \underline{66\ KN}$$

We know that, Total load on the beam = $R_A + R_B$

$$135 = R_A + 66$$

$$\therefore R_A = \underline{69\ KN}$$

Shear force diagram:

S.F at A, $F_A = +69\ KN$ (R_A)

S.F at C, $F_C = R_A - \frac{1}{2} \times 30 \times 5$

$= 69 - 75 = -6\ KN$

Shear force is constant between C and D

S.F at D, $F_D = -6 - 60 = -66\ KN$

Shear force is constant between D and E, and E and B.

S.F at B, $F_B = -66\ KN$

Bending moment diagram:

B.M at A, $M_A = 0$

B.M at B, $M_B = 0$

B.M at C, $M_C = R_A \times 5 - (\frac{1}{2} \times 30 \times 5)(\frac{1}{3} \times 5)$

$= 69 \times 5 - 75 \times \frac{5}{3}$

$= 345 - 125$

$= 220\ KNm$

BM at D, $M_D = R_A \times 6 - (\frac{1}{2} \times 30 \times 5)(\frac{1}{3} \times 5 + 1)$

$= 69 \times 6 - 75 \times (\frac{5}{3} + 1)$

$= 214\ KNm$

B.M at E, $M_E = R_A \times 8 - (\frac{1}{2} \times 30 \times 5)(\frac{1}{3} \times 5 + 3) - 60 \times 2$

$= 82\ KNm$

$M_E = 82 + 50$ – (Applied moment in cw direction)

$= 132\ KNm$

Shear force at F

$F_F = R_A - \frac{1}{2}(\frac{30}{5} x)(x)$

$0 = 69 - 3x^2$

$3x^2 = 69$

$x^2 = \frac{69}{3}$

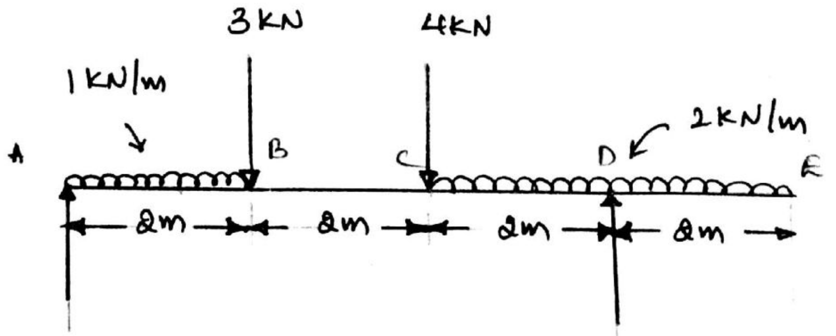
$\therefore x = 4.7958m$ [Ans]

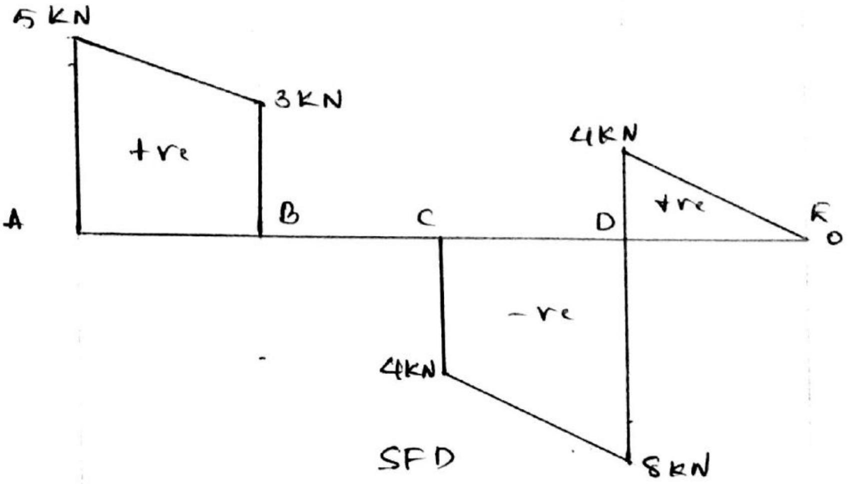
Bending moment at F

$M_F = 69 \times 4.7958 - 3 \times 4.7958^2 \times \frac{1}{3} \times 4.7958$

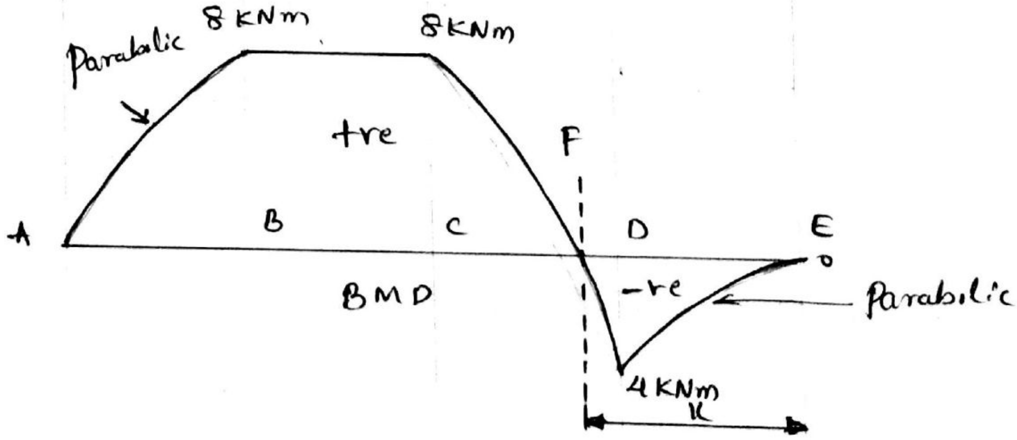
$M_F = 220.60\ KNm$ [Ans]

8.





SFD



BMD

To determine reactions at supports (R_A & R_D)

Taking moments about A, we get

$$R_D \times 6 = (2\times4)\times\left(\frac{4}{2}+4\right) + 4\times4 + 3\times2 + (1\times2)(2/2)$$

$$\therefore \quad R_D = \underline{12\,KN}.$$

Total load = $R_A + R_D$

$$(1\times2) + 3 + 4 + (2\times4) = R_A + 12$$

$$\therefore \quad R_A = \underline{\underline{5\,KN}}$$

Shear force diagram:

S.F at A, $F_A = +5$ KN

S.F at B, $F_B = 5 - 1\times2 = \underline{3\text{ KN}}$

$F_B = 3 - 3 = \underline{0\text{ KN}}$

Shear force remains constant between B and C (0 KN)

S.F at C, $F_C = 0 - 4 = -4$ KN

S.F at D, $F_D = -4 - 2\times2 = -8$ KN (without R_D)

$F_D = -4 - 2\times2 + 12 = \underline{+4\text{ KN}}$

S.F at E = $+4 - 2\times2 = 0$.

Bending moment diagram:

B.M at A, $M_A = 0$

B.M at B, $M_B = 5\times2 - (1\times2)(2/2) = +8$ KNm (Parabolic)

B.M at C, $M_C = 5\times4 - (1\times2)(2/2+2) - 3\times2 = \underline{8\text{ KNm}}$ (Constant)

B.M at D, $M_D = 5\times6 - (1\times2)(2/2+4) - 3\times4 - 4\times2 - 2\times2\,(2/2)$

$= -4$ KNm (Parabolic)

B.M at E, $M_E = 0$.

Point of contraflexure:

B.M at F, $M_F = 0 = -(2x)\left(\frac{x}{2}\right) + 12(x-2)$

$-x^2 + 12x - 24 = 0$

$x^2 - 12x + 24 = 0$

$\therefore x = + \frac{12 \pm \sqrt{12^2 - 4\times1\times24}}{2\times1}$

Taking the smaller value

$x = 2.536$m

$\therefore$ The point of contraflexure is at 2.536m from E.