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Internal Assessment Test 1 – Sept. 2017

Sub:	Dynamics of Machinery	**Code:**	15ME52
Date: 18/ 09 / 2017	Duration: 90 mins	Max Marks: 50	**Sem:** V
		Branch:	MECH

Note: Answer any **four** questions.

			Marks	OBE CO	OBE RBT
1	a	Define the following i)Sensitiveness (ii) Isochronism (iii)Hunting of governor (iv)Effort of governor	**4**	CO3	L1
	b	Derive an expression for equilibrium speed of governor	**8.5**	CO3	L3
2		The mass of each ball of a Hartnell type governor is 1.4 kg. The length of ball arm of the bell-crank lever is 100 mm where as the lengths of arm towards sleeve is 50 mm. The distance of the fulcrum of bell-crank lever from the axis of rotation is 80 mm. the extreme radii of rotation of the balls are 75 mm and 112.5 mm. The maximum equilibrium speed is 6% greater than the minimum equilibrium speed which is 300 rev/min. determine i) Stiffness of the spring and ii) Equilibrium speed when the radius of rotation of the ball is 90 mm.	**12.5**	CO3	L3
3		Explain balancing of several masses rotating in same (Graphical and Analytical Method)	**12.5**	CO2	L2
4		A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine: i. The magnitude of the masses at A and D ii. The distance between planes A and D and iii. The angular position of mass at D	**12.5**	CO2	L3
5	a	Add the following motions analytically and check graphically $x_1 = 4\cos(\omega t + 10°)$, $x_2 = 6\ \sin(\omega t + 60°)$	**8.5**	CO4	L2
	b	Explain the phenomenon of beats	**4**	CO4	L1

INTERNAL ASSESSMENT TEST - SEPT. 2017

DYNAMICS OF MACHINERY SOLUTION

1a i) **Sensitiveness** :- It is defined as the ratio of the difference between the maximum & minimum speed to the mean speed.

$$S = \frac{N_2 - N_1}{\frac{N_1 + N_2}{2}} = \frac{2[N_2 - N_1]}{N_1 + N_2}$$

ii) **Isochronous Governor** :- A governor is said to be isochronous when the equilibrium speed is constant (i.e range of speed is zero) for all radii of rotation of balls within the working range, neglecting friction.

iii) **Hunting of governor** :- A governor is said to be hunt if the speed of the engine fluctuates continuously above & below the mean speed.

iv) **Effort** :- It is the mean force exerted at the sleeve for a given percentage change of speed.

1.b. **Equilibrium Speed of governor**

Consider the forces acting on governor as shown.

Let m = Mass of each ball in kg,

M = Mass of central load in kg,

r = Radius of rotation in m,

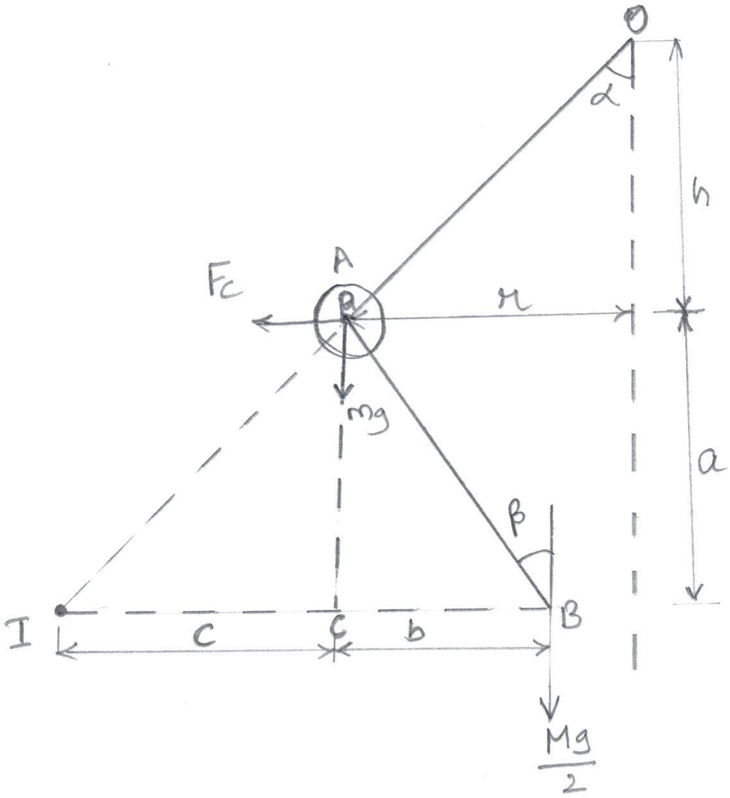
h = height of governor in m,

N = Speed of the balls in rpm,

F_c = Centrifugal force

α = Angle of inclination of upper arm to the vertical

β = Angle of inclination of lower arm to the vertical



For equilibrium $\Sigma F = 0$; $\Sigma M = 0$

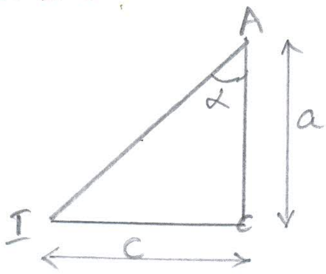
Taking moment about I

$$F_c \cdot a = mg \cdot c + \frac{Mg}{2}\left[c + b\right] \rightarrow (1)$$

$$F_c = mg \cdot \frac{c}{a} + \frac{Mg}{2}\left[\frac{c}{a} + \frac{b}{a}\right]$$

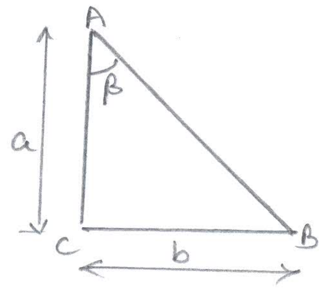
$$m\omega^2 r = mg \cdot \frac{c}{a} + \frac{Mg}{2}\left[\frac{c}{a} + \frac{b}{a}\right] \rightarrow (2) \quad \because F_c = m\omega^2 r$$

Consider Δ^{le} ACI



$\tan\alpha = \frac{c}{a} \rightarrow$ (A)

Consider Δ^{le} ACB



$\tan\beta = \frac{b}{a} \rightarrow$ (B)

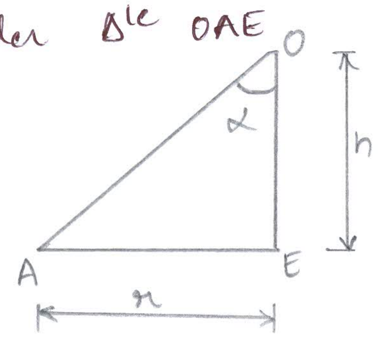
Sub. (A) & (B) in eqn (2) we get.

$$m\omega^2 r = mg.\tan\alpha + \frac{Mg}{2}\left[\tan\alpha + \tan\beta\right]$$

$$= \tan\alpha\left[mg + \frac{Mg}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

$$m\omega^2 r = \tan\alpha\left[mg + \frac{Mg}{2}(1+K)\right] \rightarrow (4) \quad \left[\because K = \frac{\tan\beta}{\tan\alpha}\right]$$

Consider Δ^{le} OAE



$\tan\alpha = \frac{r}{h} \rightarrow$ (C)

Sub. (C) in eqn (4)

$$m\omega^2 r = \frac{r}{h}\left[mg + \frac{Mg}{2}(1+K)\right]$$

$$\omega^2 = \frac{\cancel{r}}{m\cancel{r}h}\left[mg + \frac{Mg}{2}(1+K)\right]$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{1}{mh}\left[mg + \frac{Mg}{2}(1+K)\right]$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}\left[\frac{m + \frac{M}{2}(1+k)}{m}\right]$$

$$\boxed{N^2 = \frac{895}{h}\left[\frac{m+\frac{M}{2}(1+k)}{m}\right]}$$

2. <u>Given</u>

$m = 1.4\ kg$; $x = 100mm = 0.1m$; $y = 50mm = 0.05m$

$r_1 = 75mm = 0.075m$;

$r_2 = 112.5mm = 0.1125m$; $r = 0.09m$

$N_1 = 300rpm$; $N_2 = 300 + \frac{6}{100} \times 300 = 318\ rpm.$

$S = ?$; $N = ?$

<u>Angular velocity</u> : $\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi(300)}{60} = 31.42\ r/s$

<u>Centrifugal force</u>

$$F_{c_1} = m\omega_1^2 r_1$$

$$= 1.4\ (31.42)^2\ 0.075$$

$$F_{c_1} = 103.66\ N$$

<u>Angular Velocity</u> :- $\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi(318)}{60} = 33.3\ r/s$

$$F_{c2} = m\omega_2^2 r_2 = 1.4\ (33.3)\ 0.1125$$

$$F_{c_2} = 174.65\ N$$

Stiffness of Spring

$$S = 2\left[\frac{F_{c_2} - F_{c_1}}{r_2 - r_1}\right]\left[\frac{x}{y}\right]^2$$

$$= 2\left[\frac{174.65 - 103.66}{0.1125 - 0.075}\right]\left[\frac{0.1}{0.05}\right]^2$$

$$\boxed{S = 15.14 \times 10^3 \ N/m}$$

Centrifugal force at $r = 0.09m$

$$S = 2\left[\frac{F_{c_2} - F}{r_2 - r}\right]\left[\frac{x}{y}\right]^2$$

$$15.14 \times 10^3 = 2\left[\frac{174.65 - F}{0.1125 - 0.09}\right]\left[\frac{0.1}{0.05}\right]^2$$

$$F = 132.07 \ N$$

Centrifugal force

$$F = m\omega^2 r$$

$$132.07 = 1.4\left(\frac{2\pi N}{60}\right)^2 0.09$$

$$\boxed{N = 309.16 \ rpm}$$

3. Analytical Method

1. Find out the centrifugal forces exerted by each mass on rotating shaft.

2. Resolve it into horizontal & vertical components & find their sums i.e ΣH & ΣV.

Sum of horizontal components

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + \dots$$

Sum of vertical components

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + \dots$$

3. Magnitude of resultant centrifugal force

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the $\angle$le which resultant makes with horizontal

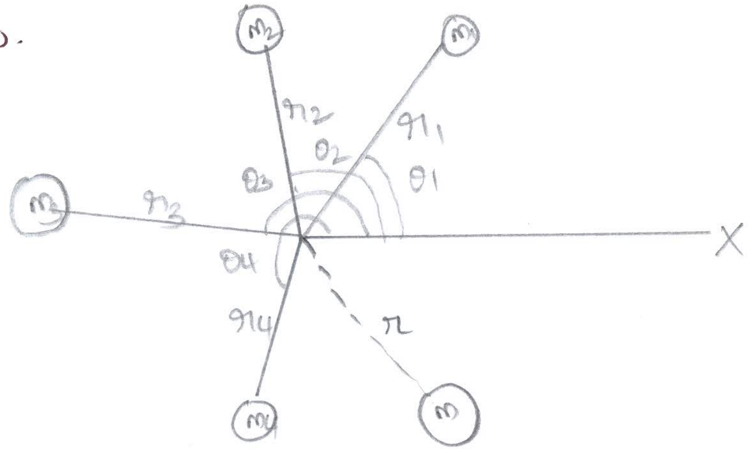
$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

5. Balancing force is equal to resultant force, but in opp. direction.

6. Find out magnitude of balancing mass at given radius.

Graphical method

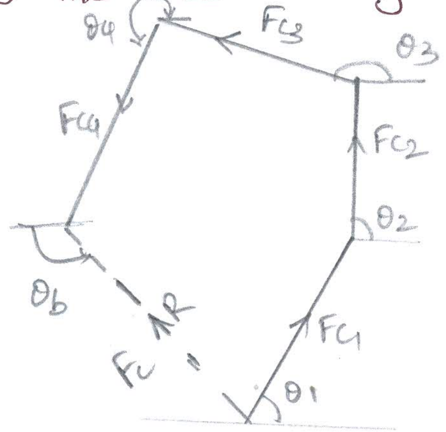
1. Draw the space diagram with the position of several masses.



Consider four masses of magnitude m_1, m_2, m_3 & m_4 at distances r_1, r_2, r_3 & r_4 from axis of rotating shaft. Let θ_1, θ_2, θ_3 & θ_4 be angles of these masses with horizontal.

2. Find out the centrifugal forces.

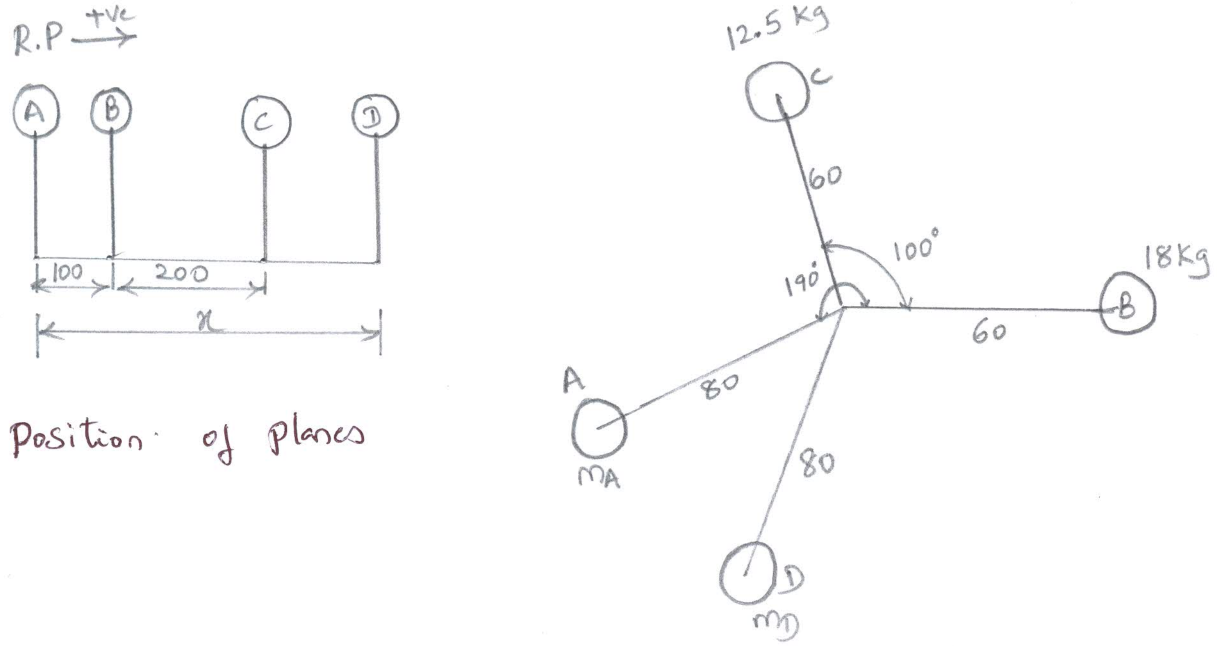
3. Draw the vector diagram with obtained centrifugal force.



4. As per polygon law, the closing side represents resulting force in magnitude & direction.

5. Balancing force is equal to resultant force, but opposite in direction.

4.



Position of planes

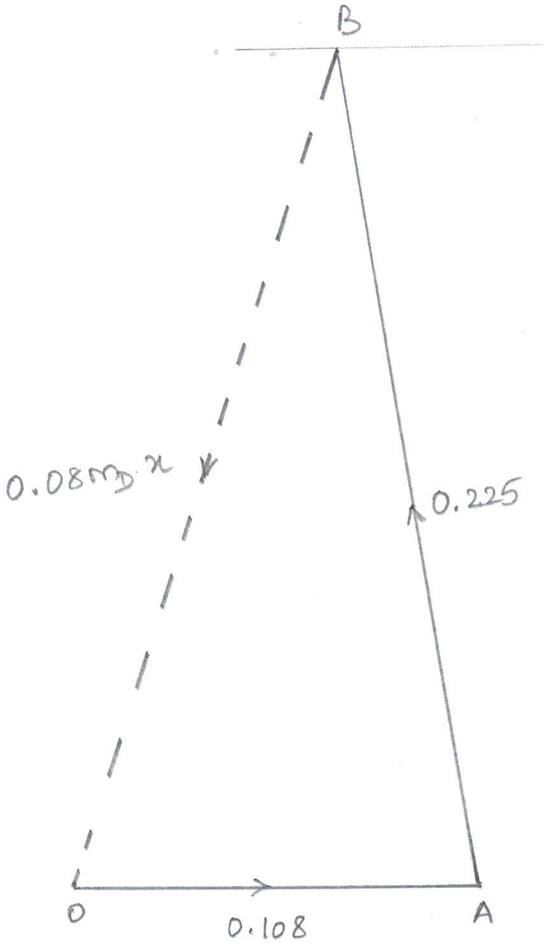
Angular position

All dimensions in mm.

Plane	Mass (m) Kg	Radius (r) m	C.F ÷ ω^2 (mr) Kg-m	Dist. from R.P (l) m	Couple ÷ ω^2 (mrl) Kg-m^2
A	m_A	0.08	$0.08\,m_A$	0	0
B	18	0.06	1.08	0.1	0.108
C	12.5	0.06	0.75	0.3	0.225
D	m_D	0.08	$0.08\,m_D$	x	$0.08\,m_D \cdot x$

Couple Polygon

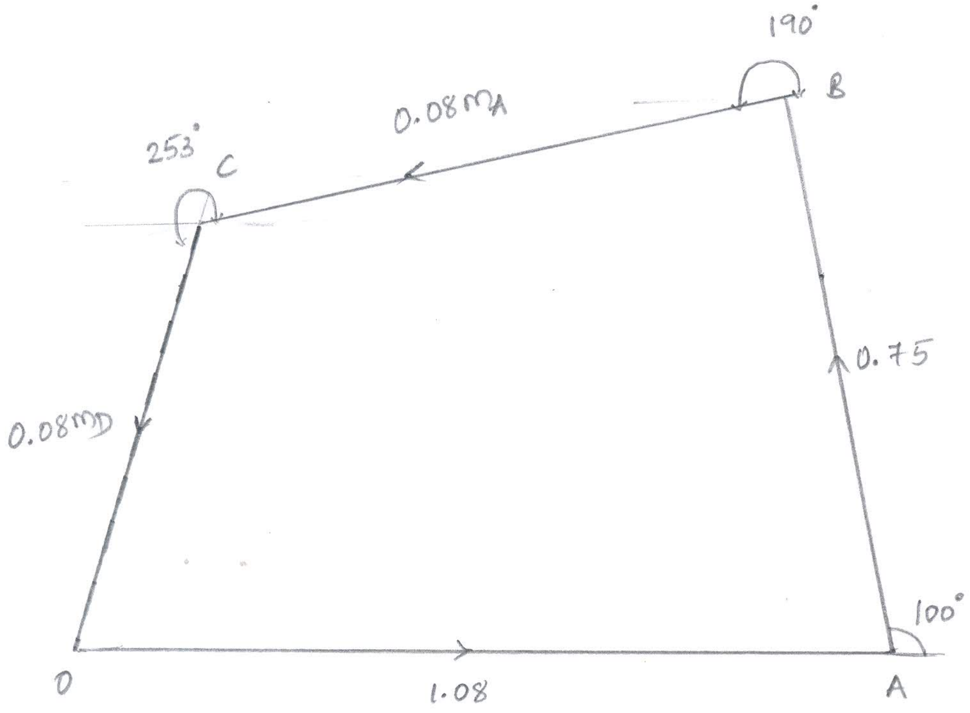
Scale 1 cm = 50 kg-m^2



$0.08\,m_D\,x = 0.232$ Kg-m^2

Force Polygon

Scale 1 cm = 10 kg-m



$0.08\, m_A = 0.79$ kg-m

$\boxed{m_A = 9.87 \text{ kg}}$

$0.08\, m_D = 0.6$

$\boxed{m_D = 7.5 \text{ kg}}$

ii) Distance b/w plane A & D.

$0.08\, m_D \cdot x = 0.232$

$0.08 \times 7.5 \times x = 0.232$

$x = 0.387$ m

$\boxed{x = 387 \text{ mm}}$

iii) Angular position of mass D is 253°

Resultant motion

5.a. $x = x_1 + x_2$

$A \sin(\omega t + \theta) = 4 \cos(\omega t + 10^\circ) + 6 \sin(\omega t + 60^\circ)$

$A \sin\omega t . \cos\theta + A \cos\omega t . \sin\theta = 4 \cos\omega t . \cos 10^\circ - 4 \sin\omega t . \sin 10^\circ + 6 \sin\omega t . \cos 60^\circ + 6 \cos\omega t . \sin 60^\circ$

$\sin\omega t (A \cos\theta) + \cos\omega t (A \sin\theta) = \cos\omega t (4 \cos 10^\circ + 6 \sin 60^\circ) + \sin\omega t (6 \cos 60^\circ - 4 \sin 10^\circ)$

$\sin\omega t (A \cos\theta) + \cos\omega t (A \sin\theta) = 9.13 \cos\omega t + 2.31 \sin\omega t .$

Equating the Coeff. of $\sin\omega t$ & $\cos\omega t$ we get.

$A \cos\theta = 2.31 \rightarrow (1)$

$A \sin\theta = 9.134 \rightarrow (2)$

Squaring & adding

$A^2 [\cos^2\theta + \sin^2\theta] = 2.31^2 + 9.134^2$

$\boxed{A = 9.42}$

Eq (2) ÷ (1)

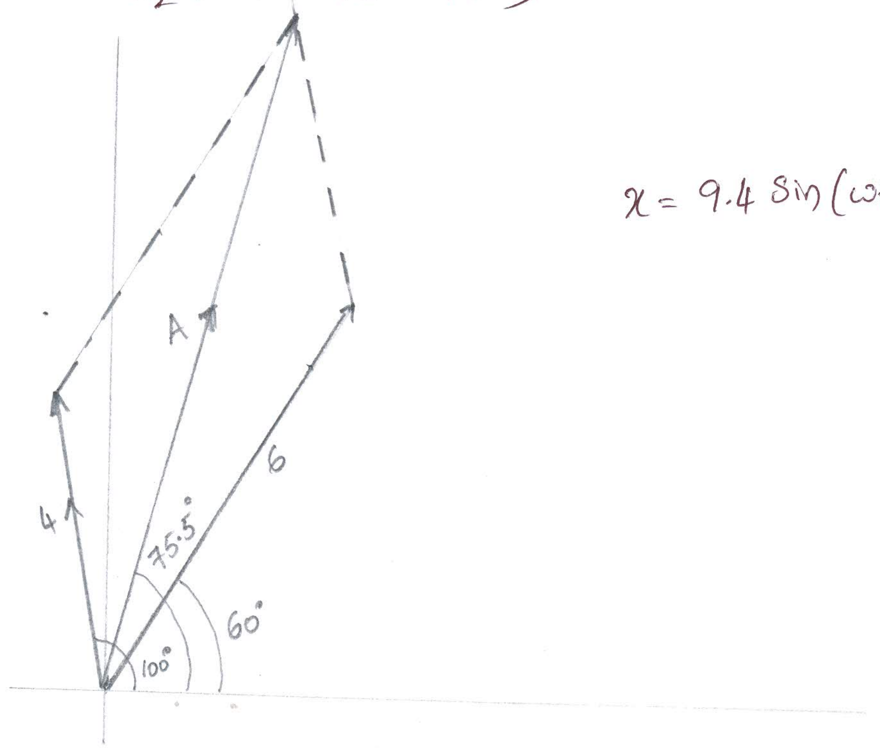
$\frac{A \sin\theta}{A \cos\theta} = \frac{9.313}{2.31}$

$\boxed{\theta = 75.8^\circ}$

$\therefore x = 9.42 \sin(\omega t + 75.8^\circ)$ //

Graphical $\quad x_1 = 4\cos(\omega t + 10^\circ) = 4\sin(\omega t + 100^\circ)$

$x_2 = 6\sin(\omega t + 60^\circ)$



$$x = 9.4\sin(\omega t + 75.5^\circ)$$

5.b. Beats

When two harmonic motions whose frequencies are close to one another are added, the resulting motion exhibits a phenomenon known as beats.

Ex:- If $x_1(t) = X\cos\omega t$

$x_2(t) = X\cos(\omega + \delta)t$

where δ is small quantity,

$$x(t) = x_1(t) + x_2(t)$$

$$= X\left[\cos\omega t + \cos(\omega+\delta)t\right]$$

$$= 2X\cos\frac{\delta t}{2} \cdot \cos\left(\omega + \frac{\delta}{2}\right)t$$

Graphical representation