1

2a. $x_1 = A\sin\omega t$, $x_2 = B\sin(\omega t+\phi)$ 1

Resulting SHM $x = x_1 + x_2$.

Expand x_1 and x_2

Substitute $A + B\cos\phi = X\cos\theta$ —— (1) 2

$B\sin\phi = X\sin\theta$ —— (2)

to get $x = X\sin(\omega t+\theta)$

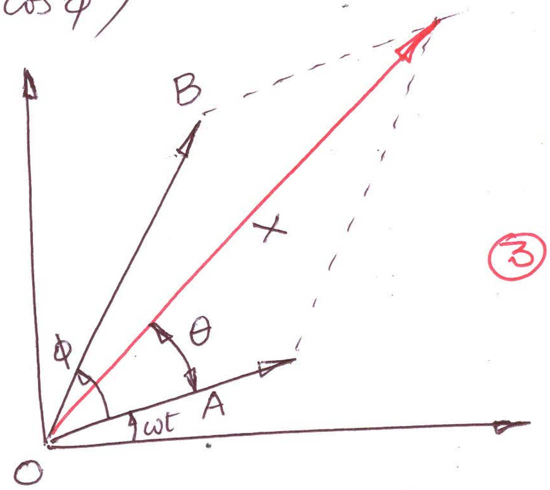
Squaring and adding (1) & (2) we get 2

$$X = (A^2 + B^2 + 2AB\cos\phi)^{1/2}$$

Dividing (2) and (1), 2

$$\theta = \tan^{-1}\left(\frac{B\sin\phi}{A+B\cos\phi}\right)$$

Graphical Representation



3

2b. $x_1 = 3\sin(8t+30^\circ)$; $x_2 = 2\cos(8t-15^\circ)$

Let the resultant be $x = A\sin(8t+\theta)$

$x = x_1 + x_2$

$A\sin 8t\cos\theta + A\cos 8t\sin\theta = 3\sin 8t\cos 30^\circ + 2\cos 8t\cos 15^\circ + 2\sin 8t\sin 15^\circ$

$3\cos 8t\sin 30^\circ +$

RHS : $\sin 8t\left[2.6+0.5176\right] + \cos 8t\left[1.5+1.932\right]$

$$= 3.1176 \sin 8t + 3.432 \cos 8t$$

Equating $\sin 8t$ and $\cos 8t$ with LHS

$$A\cos\theta = 3.1176$$
$$A\sin\theta = 3.432$$

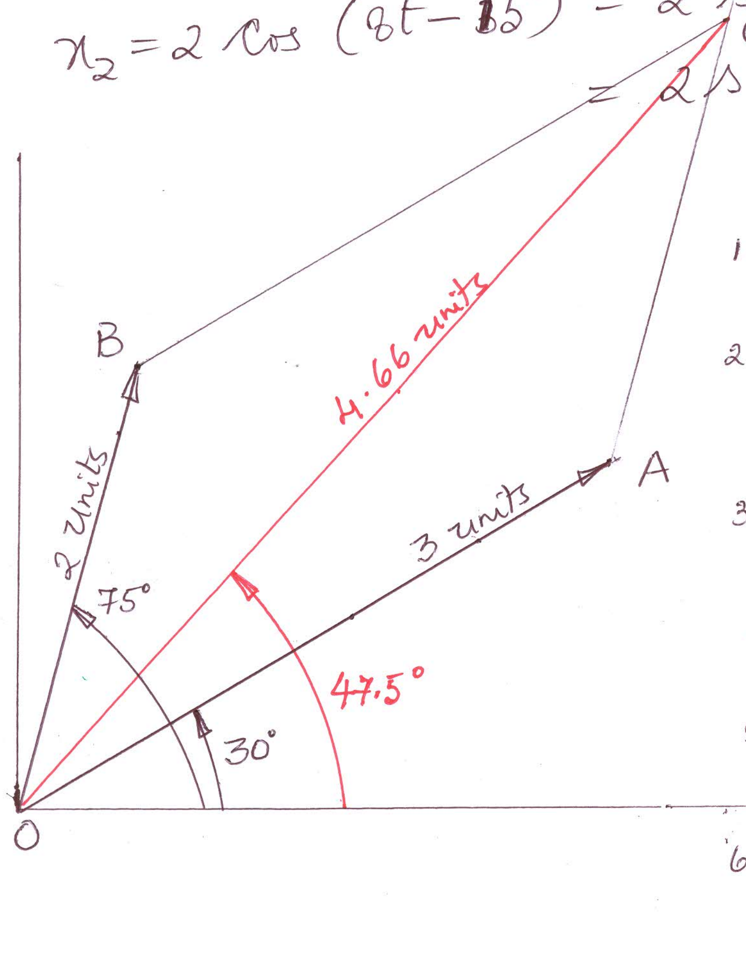
$$A = \sqrt{3.1176^2 + 3.432^2} = 4.64 \text{ units} \quad (3)$$

$$\tan\theta = \frac{3.432}{3.1176} \Rightarrow \theta = 47.748^\circ \quad (2)$$

Resultant $x = 4.64 \sin(8t + 47.748^\circ)$ (2)

Graphical (3)

$$x_1 = 3\sin(8t+30^\circ)$$
$$x_2 = 2\cos(8t-15^\circ) = 2\sin(8t-15^\circ+90^\circ)$$
$$= 2\sin(8t+75^\circ)$$



Steps:

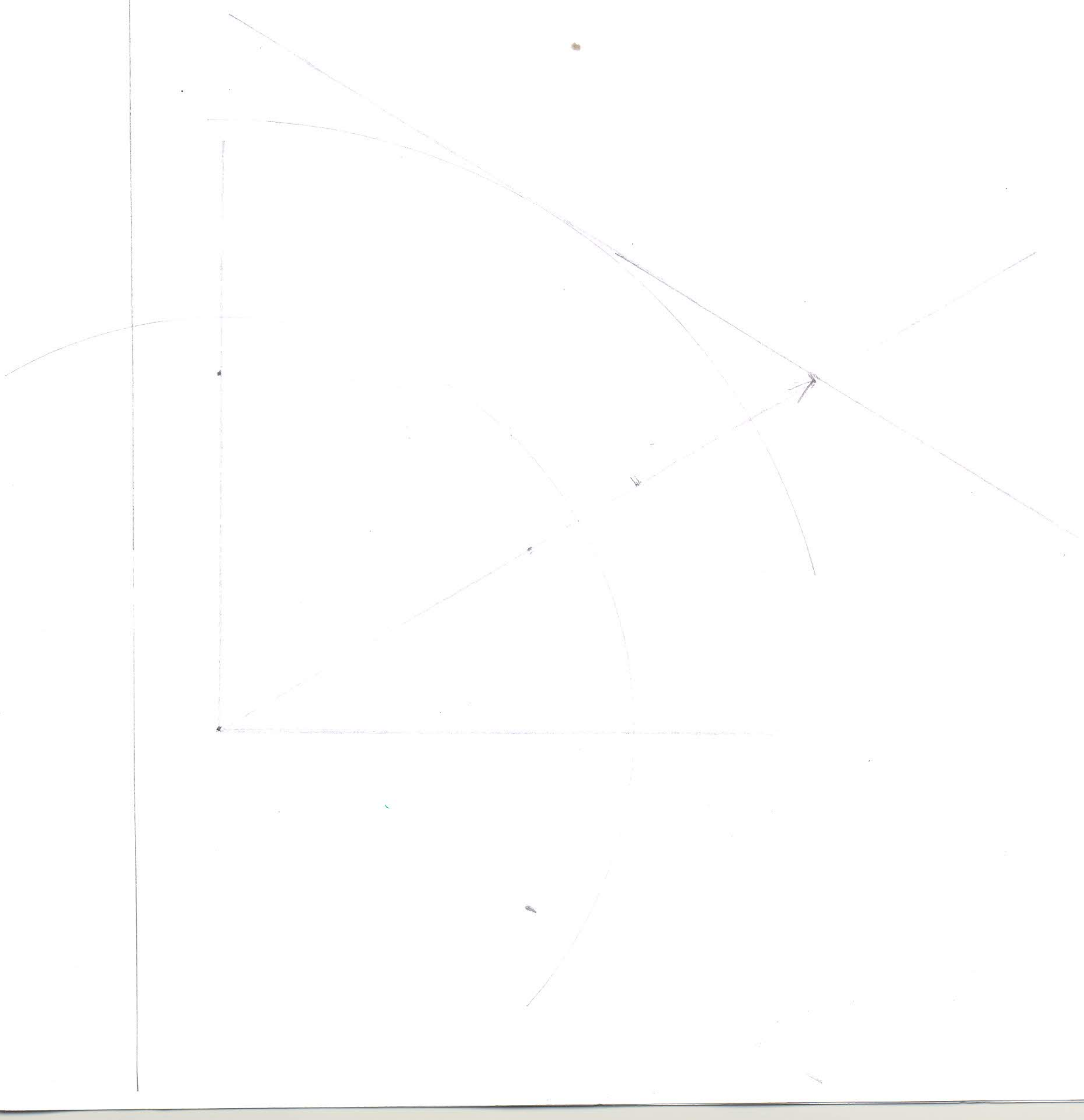
1. Express x_1, x_2 as a sine function.
2. Set-off OA = 3 units at 30° with abscissa. Choose 1 unit = 3 cm.
3. Set-off OB = 2 units at 75° with abscissa.
4. Construct a parallelogram OACB.
5. Measure OC = 14 cm or $\frac{14}{3} = 4.66$ units.
6. Measure angle OC with x-axis = 47.5°.

3a. To split $x = 50 \sin(3t + 30^\circ)$ into x_1 & x_2

such that $x_1 = A \sin(3t + 30^\circ - 60^\circ)$ and

$x_2 = 30 \sin(3t + \phi)$.

This ~~meth~~ problem can be best solved through graphical Method. For the given vectors, the solution will be indeterminate.



CMR

Find ω: Time period $= T$ sec.

$$\omega = \frac{2\pi}{T}$$

Line AB. At point A, $x(t)=0$ at $t=0$

$x(t) = 0 = m \times 0 + c$ or $c = 0$

At point B, $x(t) = +10$ at $t = T/4$

$x(t) = 10 = m \times \frac{T}{4} + c$ where $c=0$

$\therefore m = 40/T$

Equation for line AB is $x(t) = \frac{40}{T}.t$ for $0 < t \leq T/4$

(2)

Line BD At point B, $x(t) = 10$ at $t = T/4$

$x(t) = 10 = m.\frac{T}{4} + c$ —— (1)

At point D, $x(t) = -10$ at $t = 3T/4$

$x(t) = -10 = m\left(\frac{3T}{4}\right) + c$ —— (2)

(1) − (2) $\Rightarrow$ $20 = -\frac{mT}{2}$ or $m = \frac{-40}{T}$

Put in (1) $\Rightarrow$ $c = 10 - m.\frac{T}{4} = 10 - \left(\frac{-40}{T}\right)\frac{T}{4} = 20$

Equation of line BD = $x(t) = \frac{-40}{T}t + 20$ (2)

Line DE At point D, $x(t) = -10$ at $t = 3T/4$

$x(t) = -10 = m\left(\frac{3T}{4}\right) + c$ —— (3)

At point E, $x(t) = 0$ at $t = T$

$x(t) = 0 = mT + c$ or $c = -mT$.

Put $c = -mT$ in (3); $-10 = \frac{3}{4}mT - mT = -\frac{mT}{4}$ or $m = \frac{40}{T}$

But $c = -mT = -\frac{40}{T}T = -40$

Equation of line DE, $x(t) = \left(\frac{40}{T}\right)t - 40$ 2

<u>To find the coefficients of Fourier Series.</u>

$$a_0 = \frac{\omega}{2\pi}\int_0^T x(t)\,dt$$

$$= \frac{2\pi}{T}\times\frac{1}{2\pi}\left[\int_0^{T/4}\left(\frac{40}{T}\right)t\,dt + \int_{T/4}^{3T/4}\left[\frac{-40}{T}t + 20\right]dt + \int_{3T/4}^{T}\left[\frac{40}{T}t - 40\right]dt\right]$$

$$= \frac{1}{T}\left[\frac{40}{T}\left(\frac{t^2}{2}\right)_0^{T/4} + \left\{\frac{-40}{T}\left(\frac{t^2}{2}\right)_{T/4}^{3T/4} + 20(t)_{T/4}^{3T/4}\right\} + \left\{\frac{40}{T}\left(\frac{t^2}{2}\right)_{3/4T}^{T} - 40(t)_{3T/4}^{T}\right\}\right]$$

$$= \frac{1}{T}\left[\frac{40}{T}\times\frac{1}{2}\times\left(\frac{T}{4}\right)^2\right.$$

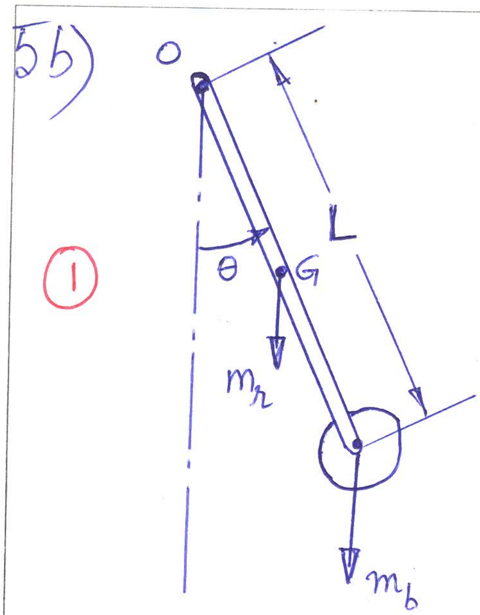
$$= \frac{1}{T}\left[\frac{5T}{4} + \left\{\cancel{-10T} + \cancel{10T}\right\} + \left\{\frac{35T}{4} - \frac{40T}{4}\right\}\right] = \frac{1}{T}\left[\frac{5T}{4} - \frac{5T}{4}\right] = 0$$

$a_0 = 0$. 2

3

CMR

5b)



1

According to Newton's II Law

$$I_0 \ddot{\theta} = \sum \text{Restoring Couples} \quad \text{——} (1)$$ 2

Here I_0 for the system is given by

$$\left(m_b L^2 + \frac{m_r}{3} \cdot L^2\right)$$ 2

Restoring Couples are

i) From the bob : $m_b g L \sin\theta$.

for small θ; $m_b \cdot g L \theta$

81

ii) From the rod :

Restoring couple is $m_r \cdot g \cdot \frac{L}{2} \sin\theta = m_r \cdot g \frac{L}{2} \theta$ for $\sin\theta \approx \theta$

2

Substituting in equation (1),

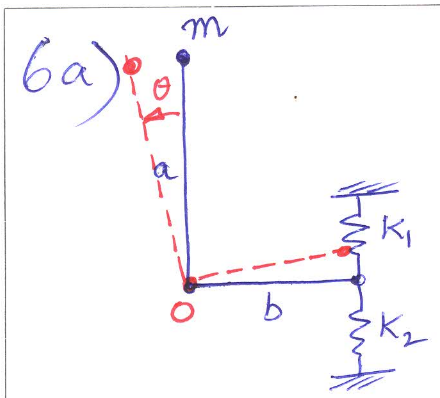
$$L^2\left(m_b + \frac{m_r}{3}\right)\ddot{\theta} = - m_b g \cdot L\theta - m_r \cdot \frac{gL}{2}\theta$$

$$\ddot{\theta}\, L\left(m_b + \frac{m_r}{3}\right) = -\left[m_b + \frac{m_r}{2}\right] g\theta$$ 2

or $$\ddot{\theta} + \frac{\left(m_b + \frac{m_r}{2}\right) g}{\left(m_b + \frac{m_r}{3}\right) L}\theta = 0$$ which is the standard equation of SHM.

Therefore $$f = \frac{1}{2\pi}\sqrt{\frac{\left(m_b + \frac{m_r}{2}\right) g}{\left(m_b + \frac{m_r}{3}\right) L}} \text{ Hz}$$ 1

6a)



Energy Method states that

$$\frac{d}{dt}(KE+PE)=0$$ for a conservative system. 1

$$KE \text{ of Mass } m = \frac{1}{2} I_0 \dot{\theta}^2$$

But $I_0 = ma^2$ or $KE = \frac{1}{2} m a^2 \dot{\theta}^2$ 1

P.E of the system

= P.E of Mass m + PE in spring K_1 + PE in spring K_2

$$= mg \cdot a\cos\theta + \frac{1}{2}K_1(b\theta)^2 + \frac{1}{2}K_2(b\theta)^2$$ 1

$$\frac{d}{dt}(KE+PE) = \frac{1}{2}ma^2(2\dot{\theta})\ddot{\theta} + \{-mga\sin\theta\} + \frac{1}{2}K_1 b^2 (2\theta)\dot{\theta} + \frac{1}{2}K_2 b^2 (2\theta)\dot{\theta} = 0$$ 2

$$= ma^2\ddot{\theta} + \{K_2 b^2 + K_1 b^2 - mga\}\theta = 0$$

Natural freq. of vibration, $f_n = \frac{1}{2\pi}\sqrt{\frac{b^2(K_1+K_2) - mga}{ma^2}}$ Hz 1

Substituting,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{0.2^2(1000+2000) + 4\times 9.81\times 0.4}{4\times 0.4^2}} = 2.19 \text{ Hz}$$

Max. Velocity = $A\omega_n = 1\cancel{0}\times\frac{\pi}{\cancel{180}\;9}\times \cancel{2}\pi \times 2.19 = 2.4$ m/s. 2

Maximum acceleration, $= A\omega_n^2 = 2.4 \times 2\pi \times 2.19$ (2)

$= 33 \ m/s^2$

6b) FBD (2)

Forces on the two springs., $mg/2$ and mg (2)

Deflection of mass, $\delta = \delta_1 + \delta_2$ (2)

Equivalent stiffness, K_e. (2)

Natural frequency $f_n = \frac{1}{2\pi}\sqrt{\frac{4K_1K_2}{m(4K_1+K_2)}}$ Hz (2)