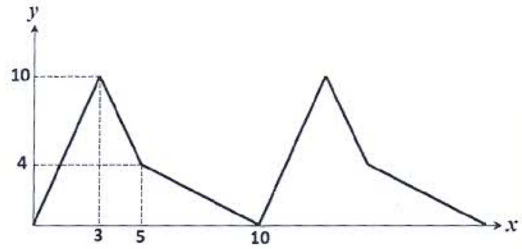
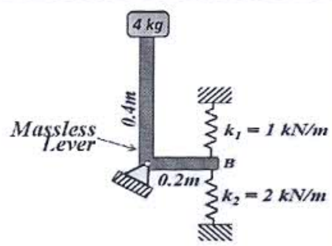
CMR INSTITUTE OF TECHNOLOGY

USN										

Internal Assessment Test I

SUBJECT: MECHANICAL VIBRATIONS								Code:	10ME72
Date:	18/09/2017	Duration:	90 min	Max. Marks:	50	Sem:	07	Branch:	MECH

Note: Answer any FIVE questions.

Q. No.		Question	Marks	OBE MAP CO	OBE MAP RBT
		PART – A			
1	A	Define (a) Simple harmonic motion (b) Frequency of free vibration (c) Degrees of freedom.	[03]	CO1	L2
	B	Sketch and label the relationship between the rotating displacement, velocity and acceleration vectors for a freely vibrating system. Show the way these relationship are obtained through the use of equations.	[03]		
	C	Evaluate ω, time period, frequency and a_o for the periodic function shown. State whether it represents a function that is odd/even/neither, and comment on the Fourier coefficients a_n and b_n for the waveform. 	[04]		
2	A	Combine the SHMs given by $x_1 = 3\ sin\ (8t + 30°)$ and $x_2 = 2\ cos\ (8t - 15°)$ analytically, and verify the result by graphical means.	[06]	CO1	L2
	B	A harmonic displacement is given by $x = 5\ sin\ (2t + \pi/6)$ where t is in seconds and phase angle in radians. Calculate (i) ω_n and the time period; (ii) $x_{max}, \dot{x}_{max}$, and $\ddot{x}_{max}$ (iii) $x, \dot{x}$, and $\ddot{x}$ at time $t = 0.3$ s.	[04]		
3	A	Derive the differential equation of motion of a spring-mass system by using the energy method.	[03]	CO2	L3
	B	Derive an expression for the natural frequency of vibration of a pendulum in which the mass of the bob is m_b; mass of the uniform rod is m_r; and the length of the rod connecting the bob and the hinge is L.	[07]		
4		Calculate the natural frequency of vibration of the system shown in the figure? Evaluate the maximum linear velocity and linear acceleration of point B when the L-shaped frame is rotated through an angle of 10° and released? 	[10]	CO2	L2
5	A	Define logarithmic decrement. Derive an expression for the logarithmic decrement. Show that $\delta = \frac{1}{n}\log_e\left\{\frac{x_0}{x_n}\right\}$	[05]	CO2	L3
	B	A 4.5 kg mass in a spring-mass-damper system executes 50 oscillations in 20 seconds. The ratio of first downward displacement to the sixth downward displacement was found to be 2.25. Calculate the stiffness of the spring and the damping force when the velocity of the mass is 1.0 m/s.	[05]		
6	A	Starting from the first principles, derive the equation of motion of a spring-mass-damper system for the case when damping factor is unity.	[06]	CO2	L3
	B	The mass of a spring-mass-dashpot system is given an initial velocity of $A\omega_n$ when the mass is at its equilibrium position. Derive the equation of motion when the damping ratio is 1.5.	[04]		

1A.

a) SHM is a type of to-and-foo motion of a particle in which the acceleration of the particle is proportional to its displacement from the mean position, and the direction of the acceleration is always opposite to the displacement direction.

1

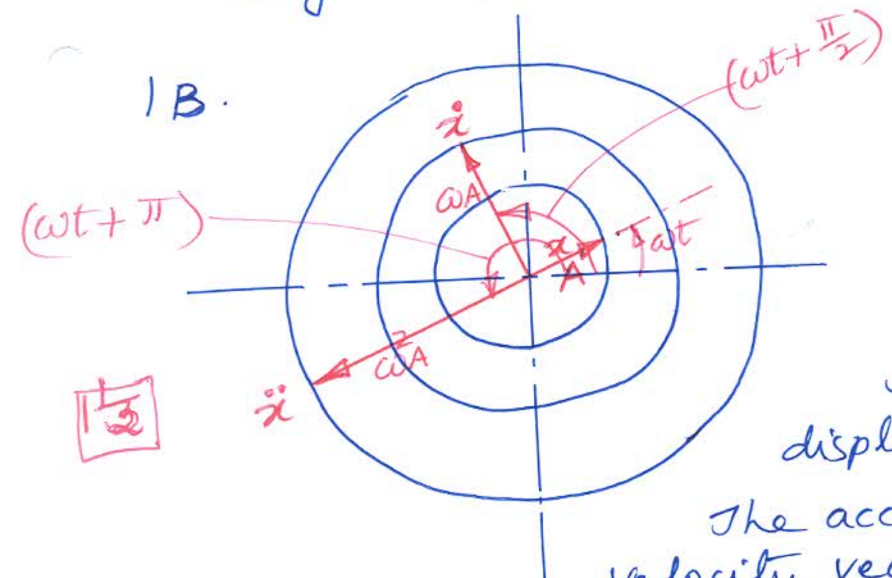
b) It is the frequency with which a body oscillates about the mean position after it is given an initial disturbance and allowed to vibrate on its own.

1

c) It is the minimum number of independent coordinates required to completely define the motion of the system at any time point.

1

1B.



$1\frac{1}{2}$

$x = A\sin\omega t$

$\dot{x} = A\omega\cos\omega t = A\omega\sin(\omega t+\frac{\pi}{2})$

$\ddot{x} = -A\omega^2\sin\omega t$

$= A\omega^2[\sin(\omega t+\pi)]$

The velocity vector leads displacement vector by 90°

The acceleration vector leads the velocity vector by 90°. The magnitudes of the displacement, velocity and acceleration vector are A, ωA and $\omega^2 A$ respectively.

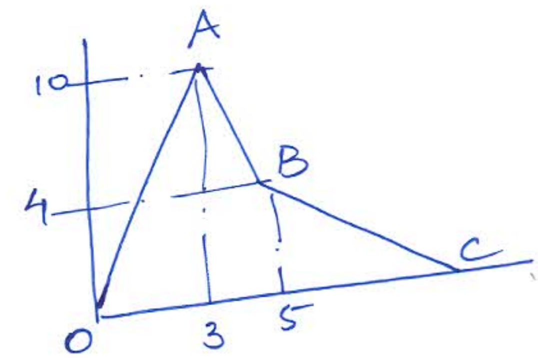
$1\frac{1}{2}$

1C. For the given waveform,

Time period $\tau = 10$ s.

Frequency $= 1/\tau = 1/10 = 0.1$ Hz.

$\omega = \frac{2\pi}{\tau} = \frac{2\pi}{10} = 0.2\pi$ rad/s



To find a_0

$$a_0 = \frac{\omega}{\pi}\int_0^{\tau} y(x)\,dx.$$

For line OA

$y = mx + c$

$O(0,0) \rightarrow \quad 0 = m \times 0 + c$ or $c = 0$

$A(3,10) \rightarrow \quad 10 = m \times 3 + c$ or $10 = 3m$ or $m = 10/3$

Eq. of OA : $\boxed{y = \frac{10}{3}x}$ for $0 \leq x \leq 3$

For line AB

$A(3, 10) \rightarrow \quad 10 = m \times 3 + c$

$B(5, 4) \rightarrow \quad 4 = m \times 5 + c$

Solving The above two equations for C,

$(10-4) = (3m - 5m)$ or $6 = -2m$ or $m = -3$

$c = 10 - 3m = 10 - 3(-3) = +19$

Eq. of AB = $\boxed{y = -3x + 19}$ for $3 \leq x \leq 5$

2

For line BC

$B(5,4) \rightarrow \quad 4 = m \times 5 + c$

$C(10,0) \rightarrow \quad 0 = m \times 10 + c$

Solving The above two equations for C,

$(4-0) = (5m - 10m)$ or $4 = -5m$ or $m = \frac{-4}{5}$

But $c = -10m = -10 \times \frac{-4}{5} = +8$

Eq. of BC = $\boxed{y = \frac{-4}{5}x + 8}$ for $5 \leq x \leq 10$

Therefore

$$a_0 = \frac{\omega}{\pi}\left[\int_0^3 \frac{10}{3}x\,dx + \int_3^5 (-3x+19)\,dx + \int_5^{10}\left(\frac{-4}{5}x+8\right)dx\right]$$

$$= \frac{\pi}{5} \times \frac{1}{\pi}\left[\frac{10}{3}\left(\frac{x^2}{2}\right)_0^3 - 3\left(\frac{x^2}{2}\right)_3^5 + 19(x)_3^5 - \frac{4}{5}\left(\frac{x^2}{2}\right)_5^{10} + 8(x)_5^{10}\right]$$

$$= \frac{1}{5}\left[\frac{10}{3} \times \frac{3^2}{2} - \frac{3}{2}\left(5^2 - 3^2\right) + 19(5-3) - \frac{4}{5\times 2}\left(10^2 - 5^2\right) + 8(10-5)\right]$$

$$= \frac{1}{5}\left[15 - 24 + 38 - 30 + 40\right] = 7.8 \quad \boxed{2}$$

2A. Combine $x_1 = 3\sin(8t + 30°)$ and $x_2 = 2\cos(8t - 15°)$

Let $x = A\sin(8t + \phi)$ be the resulting motion

$x = x_1 + x_2$ $\boxed{\frac{1}{2}}$

$$A\sin 8t\cos\phi + A\cos 8t\sin\phi = 3\sin 8t\cos 30° + 3\cos 8t\sin 30°$$
$$+ 2\cos 8t\cos 15° + 2\sin 8t\sin 15°$$

Equating terms with $\sin 8t$ term,

$$A\cos\phi = 3\cos 30° + 2\sin 15° = 3.116$$

Grouping terms with $\cos 8t$,

$$A\sin\phi = 3\sin 30° + 2\cos 15° = 3.432.$$

$$A = \sqrt{3.116^2 + 3.432^2} = 4.636. \quad \boxed{1\tfrac{1}{2}}$$

$$\tan\phi = \frac{3.432}{3.116} \text{ or } \phi = 47.76° \quad \boxed{1\tfrac{1}{2}}$$

Sum of two given vector is $4.636\sin(8t + 47.76°)$ $\boxed{\frac{1}{2}}$

Graphical $\boxed{2}$

2B. $x = 5\sin(2t + \pi/6)$

$\omega = 2$ rad/s.

$\tau = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ sec.

$A = 5$ units.

[1]

$x_{max} = A = 5$ units.

$\dot{x}_{max} = A\omega = 5 \times 2 = 10$ speed units.

$\ddot{x}_{max} = A\omega^2 = 5 \times 2^2 = 20$ accn units.

[1]

$x|_{t=0.3s} = 5\sin(2 \times 0.3 + \pi/6) = 4.51$ disp. units

$= 5 \times 0.90166$

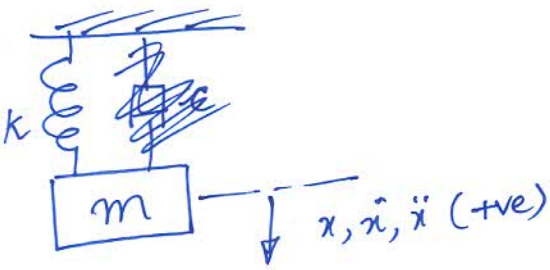
$\dot{x}|_{t=0.3s} = 5 \times 2 \times \sin(2 \times 0.3 + \pi/6 + \pi/2) = 4.324$ vel. units.

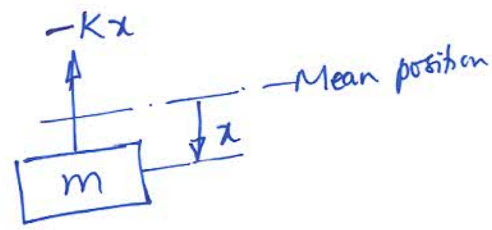
$= 10 \times 0.90166 = 9.0166$ vel. units

[2]

$\ddot{x}|_{t=0.3s} = -5 \times 2^2 \sin(0.6 + \pi/6) = -18.0332$ accn units

3A.





Energy method

For a conservative system, the total energy content of the system is constant.

That is, $\frac{d}{dt}(\text{Total energy}) = 0$

At any position x of the mass from the mean position;

The potential energy of spring $= \frac{1}{2}K \cdot x^2$

The Kinetic energy of the mass $= \frac{1}{2} m \dot{x}^2$

Total energy $TE = PE + KE = \text{Constant}$

$$\frac{d}{dt}(TE) = \frac{d}{dt}\left(\frac{1}{2}Kx^2 + \frac{1}{2}m\dot{x}^2\right) = 0$$

$$\frac{1}{2}K(2x)\dot{x} + \frac{1}{2}m(2\dot{x})\ddot{x} = 0$$

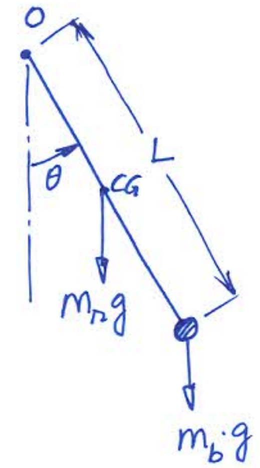
or $$m\ddot{x} + Kx = 0$$

$$\ddot{x} + \frac{K}{m}x = 0.$$

Comparing this with the equation of SHM, i.e. $\ddot{x} + \omega^2 x = 0$

we get $\omega_n = \sqrt{\frac{K}{m}}$ rad/s.

3B.



Swing the pendulum through an angle θ in anti-clockwise sense. Take CCW as +ve.

Newton's Method.

Inertia torque arising from the bob is

$$I_{o(bob)} \cdot \ddot{\theta} = m_b L^2 \ddot{\theta}$$

Inertia torque arising from the rod is

$$I_{o(rod)} \ddot{\theta} = \frac{1}{3} m_r L^2 \ddot{\theta}$$

Total Inertia torque of the system $= I_o\ddot{\theta} = \left(m_b L^2 + \frac{1}{3} m_r L^2\right)\ddot{\theta}$

Restoring torque arises from m_b and m_r. (CW torques & For small θ, $\sin\theta \simeq \theta$)

From m_b: $-m_b g(L\sin\theta) = -m_b g L\theta$

From m_r: $-m_r g\left(\frac{L}{2}\sin\theta\right) = -m_r g\frac{L}{2}\theta$

Equating Inertia & Restoring torques

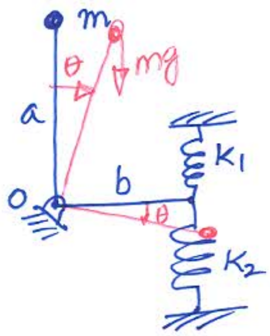
$$L\left(m_b L + \frac{1}{3} m_r L\right)\ddot{\theta} = -\left(m_b + \frac{m_r}{2}\right) g L\theta$$

$$\ddot{\theta} + \left(\frac{m_b + \frac{1}{2}m_r}{m_b + \frac{1}{3}m_r}\right)\frac{g}{L}\theta = 0$$

$$\omega_n = \sqrt{\frac{(m_b + \frac{1}{2}m_r)}{(m_b + \frac{1}{3}m_r)} \frac{g}{L}} \quad rad/s.$$

f_n = Natural frequency $= \frac{1}{2\pi}\omega_n$ Hz

4A.



Inertia torque.

Applying N's II law on the mass m which is displaced through an angle θ (cw, +ve) we get

Inertia torque $= I_0\ddot{\theta} = ma^2\ddot{\theta}$

Restoring Torque

1. From spring K_1 : $-(K_1 \cdot b\theta)\cdot b = -K_1 b^2\theta$ (minus since ccw torque)

2. from spring $K_2 = -K_2 b^2\theta$.

3. from mass m : $+mg \cdot a\sin\theta \approx +mga\theta$ (∵ cw torque)

Equating inertia & restoring torques by applying newton's law,

$$ma^2\ddot{\theta} = -b^2\theta(K_1+K_2) + mga\theta$$

$$ma^2\ddot{\theta} + \theta\left[b^2(K_1+K_2) - mga\right] = 0$$

$$\ddot{\theta} + \theta\left[\frac{b^2(K_1+K_2) - mga}{ma^2}\right] = 0$$

5

$$\therefore \omega_n = \sqrt{\frac{0.2^2(1\times10^3 + 2\times10^3) - 4\times9.81\times0.4}{4\times0.4^2}} \quad rad/s$$

$$= \sqrt{\frac{104.304}{0.64}} = 12.766 \; rad/s$$

2

$$f_n = \frac{\omega_n}{2\pi} = 2.032 \; Hz.$$

$\theta_{max} = 10^\circ = 0.17453$ radians $= A$.

$\dot{\theta}_{max} = A\omega_n = 0.17453 \times 12.766$ rad/s.
$= 2.228$ rad/s

$\ddot{\theta}_{max} = A\omega_n^2 = 28.443$ rad/s^2

$x_{max} = \theta_{max} \times 0.2 = 0.0349$ m
$\dot{x}_{max} = \dot{\theta}_{max} \times 0.2 = 0.4456$ m/s [3]
$\ddot{x}_{max} = \ddot{\theta}_{max} \times 0.2 = 5.6886$ m/s^2

5A. Logarithmic decrement definition [1]
Derivation $\delta = \log_e\left(\frac{x_1}{x_2}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$ [3]
$\delta = \frac{1}{n}\log_e \frac{x_0}{x_n}$ [1]

5B. $\delta = \frac{1}{5}\log_e \frac{x_1}{x_6} = 0.162$ [1]
$\tau_d = 0.4$ s
$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \rightarrow \xi = 0.0258$ [1]
$\omega_d = 2\pi/\tau_d = 15.71$ rad/s.
$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{15.71}{\sqrt{1-0.0258^2}} = 15.715$ rad/s [1]
$\omega_n = \sqrt{\frac{K}{m}} \rightarrow K = 1111.33$ N/m. [1]
$c_c = 2m\omega_n = 141.435$ $\frac{N.S}{m}$
$c = \xi c_c = 3.65$ NS/m.
$F_d = c\dot{x} = 3.65$ N. [1]

6A. → Set-up a spring-mass-damper system and the governing eq^n of motion for 'x' displacement.

→ Assume $x = Ae^{st}$. Differentiate to get $\dot{x}, \ddot{x}$

→ Get 2 roots s_1 and s_2

→ Express roots in terms of c_c & ξ

→ $x = A_1 e^{(-\xi+\sqrt{\xi^2-1})\omega_n t} + A_2 e^{(-\xi-\sqrt{\xi^2-1})\omega_n t}$

Case $\xi = 1$

when $\xi = 1$, $s_1 = s_2 = -\xi\omega_n = -\omega_n$ ($\because \xi = 1$)

$$x = A_1 e^{s_1 t} + tA_2 e^{s_2 t} = A_1 e^{-\omega_n t} + A_2 t e^{-\omega_n t} = (A_1 + A_2 t)e^{-\omega_n t} \quad \text{—①}$$

2

BC-1 $x(t=0) = X_0 =$ 2

BC-2 $\dot{x}(t=0) = v_0$

From ①, $X_0 = A_1$.

$$\dot{x}(t) = (A_1 + A_2 t)(-\omega_n)e^{-\omega_n t} + e^{-\omega_n t} \cdot A_2$$

$$v_0 = -A_1\omega_n + A_2 \quad \text{or} \quad A_2 = v_0 + A_1\omega_n = v_0 + X_0\omega_n.$$

$$x(t) = \left(X_0 + \{v_0 + X_0\omega_n\}t\right)e^{-\omega_n t}$$ 2

$$= \left[X_0 + \left\{\frac{v_0}{\omega_n} + X_0\right\}\omega_n t\right]e^{-\omega_n t}$$

6B. $\xi = 1.5$

$$x = A_1 e^{(-\xi+\sqrt{\xi^2-1})\omega_n t} + A_2 e^{(-\xi-\sqrt{\xi^2-1})\omega_n t}$$

Substituting $\xi = 1.5$,

$$x = A_1 e^{-0.38\omega_n t} + A_2 e^{-2.62\omega_n t}$$

But $x = 0$ at $t = 0$.

or $0 = A_1 + A_2$ —①

$$\dot{x} = A_1(-0.38\omega_n)e^{-0.38\omega_n t} + A_2(-2.62\omega_n)e^{-2.62\omega_n t}$$

Applying velocity BC,

$$\dot{x} = A\omega_n \text{ at } t=0$$

$$A\omega_n = -0.38\omega_n A_1 - 2.62\omega_n A_2$$

or $$A = -0.38 A_1 - 2.62 A_2$$

Solving the two equations,

$$A_1 = 0.446A \text{ and } A_2 = -0.446A$$

$$x = 0.446A\left(e^{-0.38\omega_n t} - e^{-2.62\omega_n t}\right)$$