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**CMR INSTITUTE OF TECHNOLOGY**

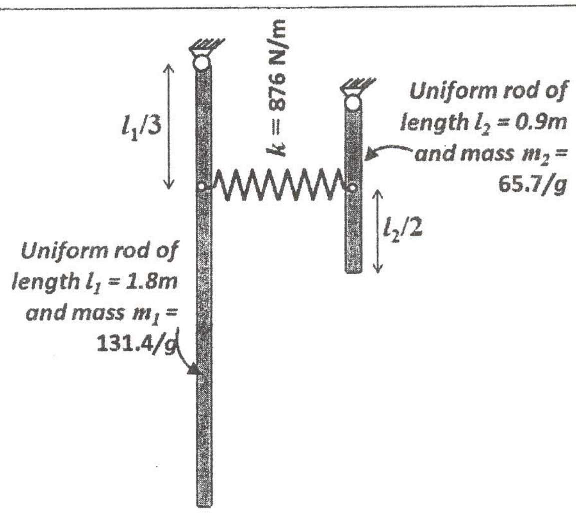
USN | | | | | | | | | | |

CMR

Internal Assessment Test II

| SUBJECT: MECHANICAL VIBRATIONS | | | | | | | | Code: | 10ME72 |
|---|---|---|---|---|---|---|---|---|---|
| Date: | 02/11/2016 | Duration: | 90 min | Max. Marks: | 50 | Sem: | 07 | Branch: | MECH |

*Note: Answer any three questions from **Part-A**. **Part-B** is compulsory. Tidiness and attention to details carry 5 marks.*

| Q. No. | | Question | Marks | OBE MAP CO | OBE MAP RBT |
|---|---|---|---|---|---|
| | | **PART – A** | | | |
| 1 | a | Starting from first principles derive the general differential equation to describe the motion of an under-damped oscillatory system. Neatly sketch the response curve. | [10] | CO2 CO3 | L3 |
| 2 | a | Derive an expression for the logarithmic decrement of a vibrating system. Following usual conventions, show that $\delta = \frac{1}{n}\log_e\left\{\frac{x_0}{x_n}\right\}$ | [6] | CO2 CO3 | L3 |
| | b | If the ratio of successive amplitudes of a viscously damped single degree of freedom system is 10:1, what will be the ratio of successive amplitudes if the damping ratio is reduced by 50%? Comment on the type of damping exhibited by the system. | [4] | CO3 | L3 |
| 3 | a | Derive an expression for the amplitude of a whirling shaft without air damping. | [4] | CO4 | L2 |
| | b | The rotor of a turbo-super charger weighing 88.3N is keyed to the centre of a 25mm diameter shaft, 40cm between the bearings. Determine (i) the critical speed of shaft; (ii) the amplitude of vibration of the rotor at a speed of 3200 rpm when the eccentricity is 0.015mm; and (iii) the vibratory force transmitted to the bearings at this speed. Assume the shaft to be simply supported and the shaft material has a density of 8gm/cm$^3$. Take E = 2.06×10$^5$MPa. | [6] | CO4 | L3 |
| 4 | a | With the help of a suitable sketch, explain dynamic vibration absorber. Show that for such systems its natural frequency should be equal to the frequency of the applied force. | [6] | CO5 | L2 |
| | b | A ring is connected to a shaft by means of a spiral spring. It is used for measuring torsional acceleration. The system is provided with a viscous damper having a spring constant of 0.12 N-ms/rad. The torsional stiffness of the spring is 1N-m/rad and the moment of inertia of the ring is 0.05kg.m$^2$. Determine the maximum acceleration of the shaft if the relative amplitude between the ring and the shaft is 2.5°. The frequency of the shaft is 20 cycles/minute. | [4] | CO4 | L3 |
| | | **PART – B** | | | |
| 5 | a | Compute the natural frequencies of the dynamic system shown.  | [15] | CO5 | L4 |

## Mapping of Course outcomes with Program Outcomes

| | Course Outcomes | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 | PO9 | PO10 | PO11 | PO12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CO1 | **Describe** the terminologies and fundamental concepts associated with a dynamical system. | 3 | | | | | | | | | | | |
| CO2 | **Discuss** the need and implications of system idealization, mathematical modelling, analysis and interpretation of the equation of the given mechanical system. | 3 | 2 | 1 | | | | | | | | | |
| CO3 | **Classify** the behaviour of a vibratory system executing free vibrations into an un-damped, under-damped, critically-damped and over-damped system, and **interpret** the response curves. | 3 | 3 | 2 | | | | | | | | | |
| CO4 | **Analyze** the given rotating system to determine the critical speed; **Recognize** the way it relates to the system response of forced vibrating system; **Interpret** the findings with reference to vibratrion measuring instruments. | 3 | 3 | 1 | | | | | | | | | |
| CO5 | **Evaluate** the natural frequencies and mode shapes of two DOF, Multi DOF systems and continuous systems, and **apply** them to practical problems. **Discuss** the principles and application of dynamic vibration absorbers. | 3 | 3 | 2 | | | | | | | | | |

| Cognitive level | KEYWORDS – Revised Bloom's Taxonomy (RBT) |
|---|---|
| L1 | List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc. |
| L2 | summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend |
| L3 | Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover. |
| L4 | Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer. |
| L5 | Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize. |

## Program Outcomes:

**PO1** - *Engineering knowledge*; **PO2** - *Problem analysis*; **PO3** - *Design/development of solutions*; **PO4** - *Conduct investigations of complex problems*; **PO5** - *Modern tool usage*; **PO6** - *The Engineer and society*; **PO7**- *Environment and sustainability*; **PO8** – *Ethics*; **PO9** - *Individual and team work*; **PO10** - *Communication*; **PO11** - *Project management and finance*; **PO12** - *Life-long learning*

Scheme of Solution IA-2 : MV : 02·Nov·2016.

1 a. (1) Physical Model of spring-mass-damper System. Force Balancing

(1) Newtons II Law : Std form of Diff Eqn.

(2) Assume Solution : Apply to Diff. Eqn ; $s_1, s_2$ ; $x(t) = x_1 e^{s_1 t} + x_2 e^{s_2 t}$

(2) $x(t) = x_1 e^{(-\xi + \sqrt{\xi^2-1})\omega_n t} + x_2 e^{\{-\xi - \sqrt{\xi^2-1}\}\omega_n t}$

(3) $x(t) = e^{-\xi\omega_n t} \times \sin(\omega_d t + \phi)$

(1) Figure.

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2a (1) Logarithmic decrement definition

(1) General expression of $x(t)$ for under-damped oscillating System

(1) Max. Amplitude for successive oscillation $(x_1)_{max}$ ; $(x_2)_{max}$. at $t = t_1, t$

(3) Ratio of successive amplitudes ; $x_1/x_n = x_1/x_2 \cdot x_2/x_3 \cdots x_{n-1}/x_n$.

$\delta = 1/n \log_e (x_0/x_n)$

2b. (1) $x_0/x_1 = 10$ ; $\delta = \log_e x_0/x_1 = 2.3$

$\delta = 2\pi\xi/\sqrt{1-\xi^2}$ ; Solve for $\xi = 0.34$

(1) Halve $\xi$ to get $\xi = 0.17$

$\log_n (x_0/x_1) = 2\pi \times 0.17/\sqrt{1-0.17^2} = 1.083$

(1) $x_0/x_1 = 2.95$

(1) Comment: Under-damped System.

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3a. (1) Sketch of the System : Defining $m, e, y, \delta, \omega, W, k$

(2) $m\omega^2(y+e) = ky$ -1

Simplify The expression to obtain $y = \pm \dfrac{e}{\frac{\omega_n^2}{\omega^2} - 1}$ -1

(1) $N_c = \frac{1}{2\pi}\omega_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = 0.4985/\sqrt{\delta}$ rps

3b. (1) Vol. of shaft $= 1.96 \times 10^{-4}\ m^3$

Mass of shaft (0.4m long) = 1.57 kg.

(3) $\delta_1$ = Static def. due to shaft weight $= 5/384\ Wl^3/EI = 1.92 \times 10^{-8}$

$I = \pi/64\ d^4 = 1.92 \times 10^{-8}\ m^4$

$\delta_2$ = Static def. due to wt. of supercharger $= \frac{WL^3}{48EI} = 2.98 \times 10^{-5}$ m.

Critical speed of shaft $= N_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{0.4985}{\sqrt{\delta_1 + \frac{\delta_2}{1.27}}} = 96.44$ rps $= 5786.9$ rpm

(1) (2) $y = \frac{0.015 \times 10^{-3}}{\left(\frac{5786.9}{3200}\right)^2 - 1} = 6.61 \times 10^{-6}$ m.

(3) $K$ = Shaft Stiffness $= \frac{48EI}{l^3} = 2966400$ N/m

(2) $\left\{\begin{array}{l}\text{Total dynamic load} \\ \text{on 2 bearings}\end{array}\right\} = Ky = 2966400 \times 6.61 \times 10^{-6} = 19.61$ N

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4 a) (2) { Sketch of the absorber + 2 diff. equation of motion.
Assume Solution for $x_1$ & $x_2$; Algebriac Equations.

(2) Expression for A and B

(2) $A_{st}$ ; $\omega_1$, $\omega_2$, $\mu$ ; Show that $A \to 0$ when $\omega = \omega_2$

4b) (1) { Circular freq. of vibrating body, $\omega = 2.0944$ rad/s
Circular freq of undamped vibration $\omega_n = \sqrt{\frac{q}{I}} = \sqrt{\frac{1}{0.05}} = 4.472$ rad/s

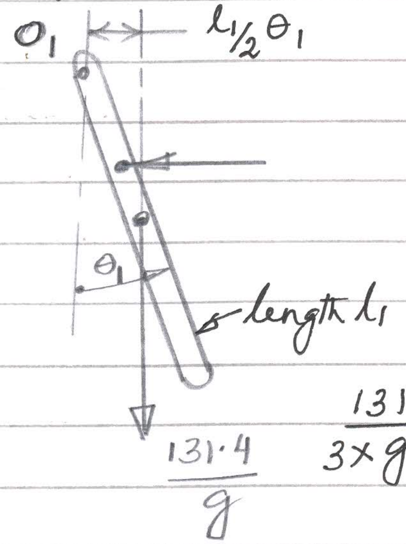
Freq. ratio $\frac{\omega}{\omega_n} = 0.4683$

(1) $c = 2I\omega_n\xi = 0.12 \Rightarrow \xi = 0.268$

(1) $\frac{\theta_z}{\theta_y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \longrightarrow \theta_y = 9.35^\circ = 0.1632$ radian

(1) Max. Angular acc$^n$ of shaft $= \omega^2 \theta_y$
$= 2.0944^2 \times 0.1632 = 0.7159$ rad/s$^2$

Q5. FBD of the rod at the LHS.




LHS Rod.

Inertia torque $= I_{O_1}\cdot\ddot{\theta}_1$ where

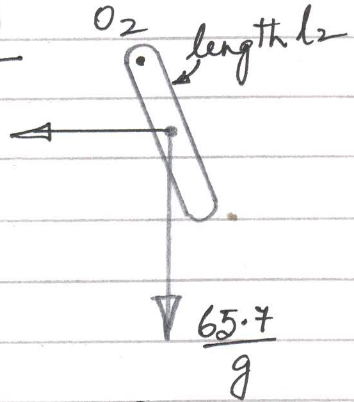
$$I_{O_1} = \cancel{mg}\cdot\frac{(131.4)}{3\times g}l_1^2$$

Restoring torque

$$= -K\left(\frac{l_1}{3}\theta_1\right)\cdot\frac{l_1}{3} + K\left(\frac{l_2}{2}\theta_2\right)\cdot\frac{l_1}{3}$$

$$\frac{131.4}{3\times g}\cdot l_1^2\ddot{\theta}_1 + K\frac{l_1^2}{9}\theta_1 - K\frac{l_1 l_2}{6}\cdot\theta_2 = 0 \quad \text{—(1)}$$

R.H.S. Rod




Inertia torque $= I_{O_2}\ddot{\theta}_2 = \frac{1}{3}\times\frac{65.7}{g}\cdot l_2^2\ddot{\theta}_2$

Restoring torque

$$= -K\left(\frac{l_2}{2}\theta_2\right)\cdot\frac{l_2}{2} + K\left(\frac{l_1}{3}\theta_1\right)\cdot\frac{l_2}{2}$$

$$\frac{65.7}{3g}l_2^2\ddot{\theta}_2 + K\cdot\frac{l_2^2}{4}\theta_2 - K\cdot\frac{l_1 l_2}{6}\theta_1 = 0 \quad \text{—(2)}$$

- Assume $\theta_1 = A\sin(\omega t)$ and $\theta_2 = B\sin\omega t$
- Convert (1) and (2) to algebriac equations in A and B
- Evaluate A and B using Cramer's rule.
- Get the frequency equation and solve for $\omega_{n1}, \omega_{n2}$