USN

10CS42

Fourth Semester B.E. Degree Examination, Dec.2017/Jan.2018

Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define the following terms and give an example for each:
i) Complete graph ii) Euler circuit iii) Path **(06 Marks)**

b. Show that in a graph G, the number of odd degree vertices is always even. **(04 Marks)**

c. Determine |V| for the following graphs:
i) G has 9 edges and all vertices have degree 3.
ii) G is registered with 15 edges.
iii) G has 10 edges with 2 vertices of degree 4 and all others of degree 3. **(06 Marks)**

d. Give pictorial and graph representation of Konigsberg bridge problem and state the problem. **(04 Marks)**

2 a. Define complete bipartite graph. Prove that Kuratowski's second graph $K_{3,3}$ is non-planar. **(06 Mark)**

b. Find the geometric dual of the graph shown in Fig.Q2(b). Write down any 4 observations of the graph and its dual.

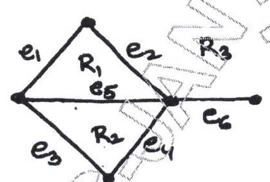

Fig.Q2(b) **(06 Marks)**

c. Find the chromatic polynomial and chromatic number for the graph shown in Fig.Q2(c).

Fig.Q2(c) **(08 Marks)**

3 a. Define a tree. In every tree T = (V, E), show that |V| = |E|+1. If a tree has 4 vertices of degree 2, 1 vertex of degree 3 and 2 vertex of degree 4 and 1 vertex of degree 5, how many pendant vertices does it have? **(06 Marks)**

b. List the vertices of the tree shown in Fig.Q3(b), when they are visited in a preorder, inorder and post order traversal.

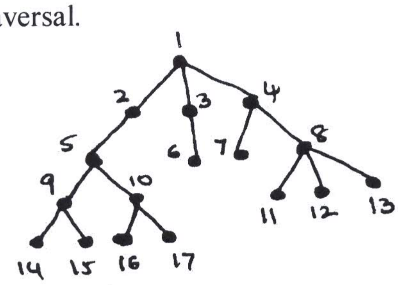

Fig.Q3(b) **(06 Marks)**

c. Obtain a prefix code to send the message "MISSION SUCCESSFUL" using labeled binary tree and hence encode the message. **(08 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

4 a. Define the following terms and give an example for each:
i) Cutset ii) Edge connectivity iii) Complete matching **(06 Marks)**

b. Table.Q4(b) summarizes the friendships between four girls g_1, g_2, g_3, g_4 and five boys b_1, b_2, b_3, b_4, b_5. Prove that each girl can marry a boy who is her friend. **(06 Marks)**

Girl	Boy friend
g_1	b_1 b_4 b_3
g_2	b_1
g_3	b_2 b_3 b_4
g_4	b_2 b_4

Table.Q4(b)

c. Bring out major steps in Prim's algorithm and find the shortest spanning tree of a weighted graph shown in Fig.Q4(c).

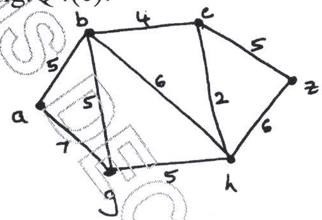

Fig.Q4(c) **(08 Marks)**

PART – B

5 a. Find the number of arrangements of the letters in TALLAHASSEE which have no adjacent A's. **(05 Marks)**

b. Find the term which contains x'' and y^4 in the expansion of $(2x^3 - 3xy^2 + z^2)^6$. **(05 Marks)**

c. How many positive integers n can be formed using the digits 3 4 4 5 5 6 7 if we want n to exceed 5,000,000? **(05 Marks)**

d. Define Catalan number. In how many ways can one arrange 3 1's and 3 −1's so that all 6 partial sums (starting with the 1st summand) are non-negative? List all the arrangements. **(05 Marks)**

6 a. Using the principle of inclusion and exclusion, determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2 or 3 or 5. **(06 Marks)**

b. Define derangement. There are 8 letters to 8 different people to be placed in 8 different addressed envelopes. Find the number of ways of doing this so that at least one letter gets to the right person. **(06 Marks)**

c. A girl has sarees of 5 different colors – blue, green, red, white and yellow. On Monday, she does not wear green, on Tuesday blue or red, on Wednesday blue or green, on Thursdays red or yellow, on Friday red. In how many ways can she dress without repeating a color during a week (from Monday to Friday)? **(08 Marks)**

7 a. Find the coefficient of x^{18} in the product $(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + ...)^5$. **(05 Marks)**

b. Find the exponential generating function for the number of way to arrange 'n' letters, $n \ge 0$, selected from each of the following words: i) HAWAII, ii) MISSISSIPPI, iii) ISOMORPHISM. **(05 Marks)**

c. In how many ways can 12 oranges be distributed among three children A, B and C so that A gets atleast 4, B and C get atleast 2 but C gets no more than 5? **(05 Marks)**

d. Find the number of partitions of positive integer n = 6 into distinct summands as a coefficient of x^6 in the generating function of $P_d(6)$. Also list these partitions. **(05 Marks)**

8 a. Solve the recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ given $a_0 = 1$, $a_1 = 4$, $a_2 = 28$. **(06 Marks)**

b. Solve the following recurrence relation using the method of generating functions:
$a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \ge 0$, $a_0 = 3$, $a_1 = 7$ **(08 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**